

# Predicting Flux Rates against Pressure via Solution-Diffusion in Reverse Osmosis Membranes

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**Abstract-**This paper suggests a new method of predicting flux values at Reverse Osmosis (RO) desalination plants. The solution-diffusion model is utilized to determine the osmotic pressure drops for seawater sources. The same technique was applied to the groundwater source at the Abqaiq plant (500 RO plant) to calculate the osmotic pressure. The calculated osmotic pressures were utilized to determine the appropriate flux rates and membrane resistances of different BWRO Toray membranes and a performance comparison between various membranes has been established. The model results confirm an inverse relationship between membrane thickness and water flux rate. Also, a proportional linear relation between the overall water flux and the applied pressure is identified. Higher flux rates and lower salinity indicate lower membrane resistance yielding higher production. The modeled data predict that BWRO Toray TM720D-440 with an 8" membrane is the optimal choice for treating waters from the three water sources at the Abqaiq plant.

*Keywords-*reverse osmosis; treatment; desalination; modeling

## I. INTRODUCTION

The solution-diffusion model is a popular expression used to explain the transport in dialysis, reverse osmosis, gas permeation, and pervaporation. Previous experimental data and modeling results verified that the flux rate is proportional to the gradient in the chemical potential [1]. There are two different models that describe and control the permeation in membranes for better separation. The first model is the solution-diffusion model where permeants dissolve (sorption) in the membrane material at the upstream interface in the presence of a concentration gradient that allows permeants to diffuse through the membrane and desorb on the downstream interface side. The separation between different permeants occurs because each material has a different diffusion rate in the membrane. The solution-diffusion model has been used since 1940 to explain the transport of gases across polymeric membranes. A second model, called the pore-flow model, depends on the presence of a pressure gradient for a convection flow of permeants through the membrane's tiny pores, and is more limited compared to the first model. Exclusion or filtration of larger permeant's pores is the separation technique explained via the pore-flow model [1, 2]. There is a major difference between the solution-diffusion model and the pore-flow model in expressing the chemical potential. In the solution-diffusion

model, the pressure within a membrane is uniform and the chemical potential gradient is expressed only as a concentration gradient. Solution-diffusion membranes transmit pressure in the same way as liquids, which is the reason for expressing the pressure difference across the membrane as a concentration gradient only. On the other hand, the chemical potential gradient in the pore-flow model is expressed only as a pressure gradient since the concentrations of both solvent and solute within a membrane are uniform. Comparisons between the two models for a one-component solution in a pressure-driven permeation system were conducted in [1, 2].

The objective of this work is to estimate the osmotic pressure drop value of the high rejection brackish water RO membrane (Toray TM720D-400 with 8") by using the solution-diffusion model that is applied to the Abqaiq plant (500 RO plant) for Shedgum/Abqaiq groundwater at Saudi Aramco, Dhahran, Saudi Arabia. Osmotic pressure drops have been calculated for the groundwater, the Arabian Gulf water, and the Red Sea water at the same plant configuration and operating conditions of the Abqaiq plant in Aramco. The calculated osmotic pressures are utilized to determine the applied pressure drop across the membrane and the applicability of using different BWRO Toray membrane types for the treatment of seawater. The maximum achievable water flux values are determined for the various suggested BWRO membranes for the three water sources. Also, the membrane resistance values have been investigated for comparison purposes. The ideal membrane for the treatment of various water sources at a RO plant with the same configuration of the Abqaiq plant has been selected. The feasibility of using BWRO membranes in the desalination of Red Sea water in Jeddah, Saudi Arabia is studied at the same flux rate of the Arabian Gulf water source and the same plant conditions. Osmotic pressure drop, applied pressure drop, flux rates, and membrane resistance values for the Red Sea water source were compared with those of Shedgum/Abqaiq groundwater and Arabian Gulf water.

## II. REVERSE OSMOSIS

In reverse osmosis, water flows from the salt solution to the pure waterside by applying pressure ( $\Delta p$ ) that is greater than the osmotic pressure ( $\Delta \pi$ ) [1]. Generally, in reverse osmosis, the condition  $\Delta p > \Delta \pi$  must be satisfied all the time to allow water to pass through the membrane and reach the permeate

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side [1, 2]. Reverse osmosis membranes are preferred over ultrafiltration and nanofiltration since they are capable of removing 90 to 99% of TDS in water [3]. The osmotic pressure  $\Delta\pi$  is defined as the pressure difference  $p_o - p_f$  across the membrane. If a pressure higher than the osmotic pressure is applied to the feed side of the membrane [1]. This process is called the reverse osmosis. The driving forces in a reverse osmosis membrane according to the solution-diffusion and the pore-flow models are visualized in [1].  $\mu_i$  and  $\gamma_i$  are the chemical potential and activity coefficient, respectively, of component  $i$  [1].

### III. METHOD AND EQUATIONS

Collected Abqaiq 500 RO plant data (Table I) have been used to determine osmotic pressure drop values for the RO membrane (Toray TM720D-400 with 8") from (1) and (2). However, in order to calculate the osmotic pressure for seawater sources, the same information of Shedgum/Abqaiq groundwater at Abqaiq 500 RO plant was applied, except for the flux and salinity values, for the treatment of either the Arabian Gulf or the Red Sea waters as listed in Table I [1, 4]. Water permeability is approximately determined to be  $9.5 \times 10^{-7} \text{ cm}^2/\text{s}$  [8]. For water-salt solution, reverse osmosis permeation expression can be simplified as [1, 5]:

$$J_i = A(\Delta p - \Delta\pi) \quad (1)$$

$$A = \frac{P_i c_{io} v_i}{RT\ell} \quad (2)$$

where  $J_i$  is the membrane flux of component  $i$ , water (gfd),  $\Delta p$  is the applied pressure drop across the membrane (psi),  $\Delta\pi$  is the osmotic pressure drop across the membrane (psi),  $A$  is the water permeability constant ( $\text{cm}^2/\text{atm}\times\text{s}$ ),  $P_i$  is the permeability of component  $i$ , water ( $\text{cm}^2/\text{s}$ ),  $c_{io}$  is the initial mole concentration of water (ppm),  $v_i$  is the water molar volume ( $\text{cm}^3/\text{mol}$ ),  $T$  is the water temperature (K),  $R$  is the gas constant, and  $\ell$  is the membrane thickness which is assumed to be similar to spacer thickness (mil).

Membrane resistance [8] constants for each BWRO Toray membrane have been calculated by using (3).

$$J_i = \frac{\Delta p}{\kappa \mathcal{R}_m} \quad (3)$$

where  $J_i$  is the membrane flux of component  $i$ , water (gfd),  $\Delta p$  is the applied pressure across the membrane (psi),  $\kappa$  is the dynamic viscosity of water ( $\text{lb s}/\text{ft}^2$ ), and  $\mathcal{R}_m$  is the membrane resistance (1/ft).

$$\pi = \mathcal{M}RT \quad (4)$$

In (4),  $\mathcal{M}$  is the molar concentration of dissolved species (mol/L),  $R$  is the ideal gas constant ( $0.08206 \text{ L}\cdot\text{atm}/\text{mol}\cdot\text{K}$ ), and  $T$  is the water temperature (K).

Equation (5) defines the ability of a membrane to separate salt from the feed solution which is known as membrane removal percentage ( $\chi$ ) and it increases with the applied pressure. The feed TDS concentration is taken from the three studied sources, as shown in Table I, while the outlet TDS concentration is determined by using (5) at a similar removal percentage of Toray TM720D-400 with 8" membrane that is 99.8% (Table IV). The water molecular weight (18g/mol) should be used to convert the ppm values to molar concentrations of TDS.

$$\chi = \left( \frac{c_{jo} - c_{je}}{c_{jo}} \right) \times 100 \quad (5)$$

where  $\chi$  is the membrane removal percentage,  $c_{jo}$  is the initial concentration of component  $j$ , salt (ppm), and  $c_{je}$  is the final concentration of component  $j$  (ppm).

Table II shows the applied pressure drop must be at 20psi or below per element (RO module) and 60psi or below per vessel [4, 6]. The assumption of having equal pressure on membranes per vessel would simplify our calculations. Altaee's study showed that permeate flow, pressure and recovery rate are distributed almost equally to membranes per RO vessel [10]. A field study confirmed an improved performance by rearranging the elements in pressure vessels in order to reduce the pressure drop and permeate conductivity across the vessel [11]. Typical flux rates and maximum recovery values for the groundwater and the two studied water source scenarios (the Arabian Gulf and the Red Sea waters) at Abqaiq 500 RO plant are given in Table III.

TABLE I. DATA OF RO MEMBRANE PROCESS AT ABQAIQ 500 RO PLANT AND THE TWO SEAWATER STUDIED SCENARIOS [1, 4, 6, 7]

Parameter	Shedgum/Abqaiq groundwater	Arabian Gulf water	Red Sea water
Membrane type	Toray TM720D-400 with 8"		
RO module	72 parallel membranes $\times$ 8 units		
Membrane thickness ( $\ell$ ) [3]	Assumed to be similar to spacer thickness of 34 mil		
Membrane area (Area) [3]	400ft <sup>2</sup>		
Max pressure drop per vessel ( $\Delta p$ )	~60psi		
Max pressure drop per membrane ( $\Delta p$ )	~20psi		
Water salinity ( $c_{io}$ )*	~2800 [4]	~41070 [6]	~42070 [7]
Membrane water flux ( $J_i$ )*	~18gfd	~12 gfd	~12 gfd
Water temperature ( $T$ )	25C		
Water permeability constant ( $P_i$ )**	$9.5 \times 10^{-7} \text{ cm}^2/\text{s}$		
Water molar volume ( $v_i$ )	18cm <sup>3</sup> /mol		
Gas constant ( $R$ )	$8.2057 \times 10^{-5} \text{ m}^3 \cdot \text{atm}/\text{mol}\cdot\text{K}$		

\*Averaged values

\*\* Taken from [2], regardless of the temperature effect on permeability. Can be calculated at different temperatures [16]

TABLE II. OPERATING DESIGN LIMITS OF THE OVERALL RO MODULE AT ABQAIQ 500 RO PLANT [4, 12, 13]

Operating limits	
Maximum operating pressure	600psi (4.1MPa)
Maximum feed water temperature	113°F (45°C)
Maximum feed water SDI15	5
Feed water chlorine concentration	Not detectable
Feed water pH range, continuous operation	2-11
Feed water pH range, chemical cleaning	1-12
Maximum pressure drop per element	20psi (0.14MPa)
Maximum pressure drop per vessel	60psi (0.4MPa)

TABLE III. CHARACTERISTICS OF GROUNDWATER SOURCE AND STUDIED WATER SOURCES AT THE ABQAIQ 500 RO PLANT [4]

Water source	Shedgum/Abqaiq groundwater	Arabian Gulf	Red Sea
Feed silt density index	SDI < 3	SDI < 4	SDI < 4
Typical target flux (gfd)	18	12	12
Max. element recovery (%)	19	14	14

The determined osmotic pressure values for the RO membrane (Toray TM720D-400 with 8") of the groundwater and the two studied water sources are used again in (1) to calculate the applied pressure drop and suggested flux values. The same osmotic pressure drop for each case is utilized to determine the results of different Toray BWRO membrane types at high, low, and standard operating pressure as shown in Table IV. TM-720-370 and TM720-440 are standard BWRO membranes and TM720C-440, TM720L-400, and TM720L-440 are low-pressure BWRO membranes whereas TM720DA400, TM720D-400, and TM720D-440 are high-pressure BWRO membranes. It is worth mentioning that our applied pressure drop must be higher than the calculated osmotic pressure in order to have a positive flux. TS-diagrams [7] are used to determine the exact value of water densities at different feed sources from the average water temperature and water salinity (Table V). The exact water densities allow us to convert gas constant values from  $m^3 \cdot atm/mol \cdot K$  to  $kg \cdot atm/mol \cdot K$  to progress calculations.

TABLE IV. TORAY BRACKISH WATER RO 8" DIAMETER MEMBRANES [13, 14]

Category	Type	Rejection (%)	Thickness (mil)*
Standard BWRO	TM720-370	99.7	31
	TM720-440	99.7	28
High-pressure BWRO	TM720DA400	99.8	31
	TM720D-400	99.8	34
	TM720D-440	99.8	28
Low-pressure BWRO	TM720C-440	99.2	28
	TM720L-400	99.5	31
	TM720L-440	99.5	28

\* The membrane thickness is assumed to be the same as spacer thickness

TABLE V. WATER DENSITIES FROM TS-DIAGRAMS [4, 6, 7, 15]

Water Source	Temperature (°C)	Salinity (ppm)	Density ( $kg/m^3$ )
Shedgum/Abqaiq groundwater	25	2800 [2]	999.19
Arabian Gulf	25	41070 [4]	1027.97
Red Sea	25	42070 [6]	1028.67

## IV. RESULTS AND DISCUSSION

Equations (1) and (2) allow us to calculate the osmotic pressure drop for each water source (Table VI). The osmotic pressure of the groundwater source is less than the Arabian Gulf and the Red Sea water sources which is related to the flux rates and water salinity. Flux rates for the Arabian Gulf and the Red Sea waters are approximately the half of the groundwater source. However, the water salinity of the groundwater source is much lower than the other sources. Therefore, the required applied pressure drop must be larger in the case of seawater sources due to their higher determined osmotic pressure values. Since the plant configuration has 8 elements per vessel, we should have a maximum osmotic pressure of 60psi or less per vessel which is equivalent to a max pressure of 7.5psi per membrane, assuming that the pressure is distributed equally on membranes per vessel. The selected applied pressure range for our study is 6.5 to 7.5psi. Maximum pressure values are assigned to the different membranes based on their category as illustrated in Table VII.

TABLE VI. CALCULATED OSMOTIC PRESSURE DROP ( $\Delta\pi$ ) FOR EACH WATER SOURCE

Water source	A (cm/atm.s)	$J_i$ (cm/s)	$J_i/A$ (atm)	$\Delta\pi$ (atm)	$\Delta\pi$ (psi)	$\Delta\pi$ per vessel < 60 (psi)
Shedgum/Abqaiq groundwater	0.00808	0.00083	0.1028	0.44	6.48	51.84
Arabian Gulf	0.00755	0.00056	0.0742	0.47	6.90	55.21
Red Sea	0.00754	0.00056	0.0743	0.47	6.90	55.20

TABLE VII. ASSIGNED PRESSURE VALUES FOR TORAY BWRO MEMBRANES

Category	Type	$\Delta p$ range (psi)*
Standard BWRO	TM720-370	6.50 - 7.25
	TM720-440	6.50 - 7.25
High-pressure BWRO	TM720DA400	6.50 - 7.50
	TM720D-400	6.50 - 7.50
	TM720D-440	6.50 - 7.50
Low-pressure BWRO	TM720C-440	6.50 - 7.00
	TM720L-400	6.50 - 7.00
	TM720L-440	6.50 - 7.00

\* High and low-pressure values are taken relative to the standard pressure

The relationship between the applied pressure drops and the overall water flux rates for the groundwater source are obtained in Figure 1(a)-(c) for standard, high-pressure, and low-pressure Toray BWRO membranes. Figure 1 shows that the maximum possible flux for the groundwater in the standard membranes is around 11gfd for TM720-440 membrane. In Figures 3(b) and 3(c) the highest observed groundwater flux in the high-pressure and low-pressure membranes are 14.7gfd for TM720D-440 and 7.5gfd for TM720C-440 and TM720L-440 respectively (blue and green lines overlap). This observation is associated with the membrane thickness in which the least membrane thickness (28mils) has been capable to achieve the highest flux. This confirms an inverse relationship between the membrane thickness and the water flux rate. Further, there is a linear relationship between the applied pressure drop and the overall water flux.

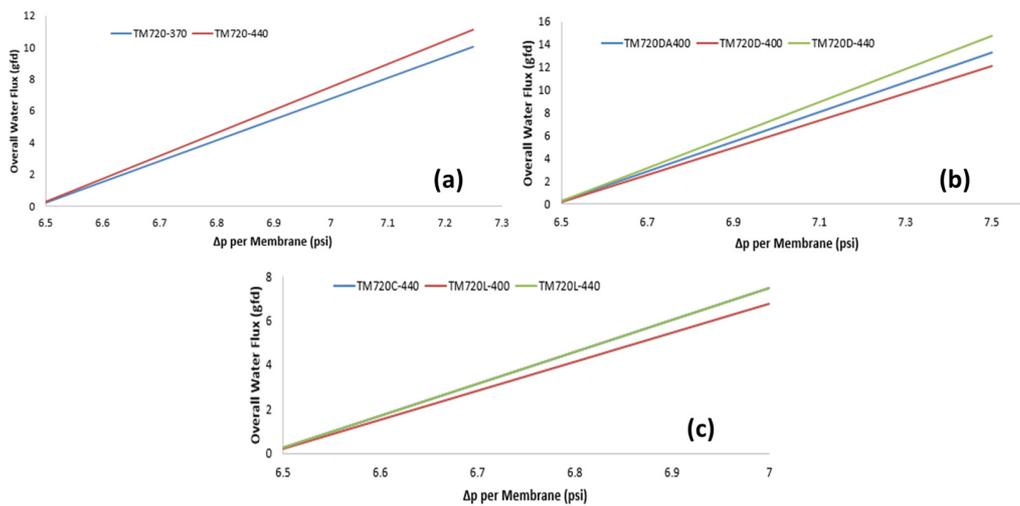


Fig. 1. Effect of different applied pressures on the groundwater flux for (a) Toray standard BWRO membranes, (b) Toray high-pressure BWRO membranes, and (c) Toray low-pressure BWRO membranes.

Figure 2 identifies a proportional relationship between the water flux and the applied pressure across the membrane. The highest recorded flux is accounted for TM720D-440 for Shedgum/Abqaiq groundwater because water TDS is low for groundwater and TM720D-440 has the lowest thickness and the highest pressure range. The Arabian Gulf and the Red Sea water sources almost have similar flux rates at the same applied pressures due to the similarities in their water salinity levels. TM720C-440, TM720L-400, and TM720L-440 membranes reserved the lowest flux values since they are categorized as low-pressure BWRO membranes.

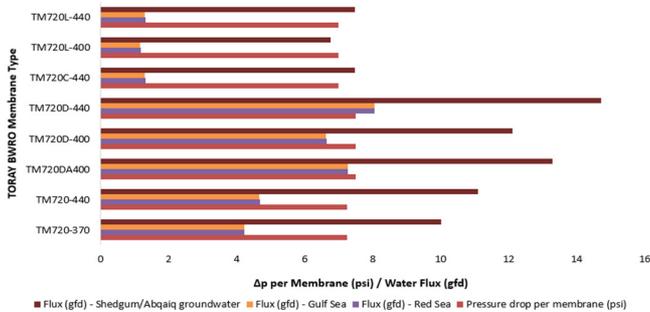


Fig. 2. Observed water flux for various water sources at different applied pressures using Toray BWRO membranes.

Figure 3 demonstrates the membrane resistance for the three studied water sources. Seawater sources have higher membrane resistances than the groundwater source because of their lower flux and higher TDS. TM720L-400 has the highest membrane resistance since it is in the low-pressure category and has the highest membrane thickness of 31mils. Equation (4) calculations are shown in Table VIII. The study predictions estimated that the overall osmotic pressure drops required for seawater and groundwater treatment plants are approximately 55psi and 830psi respectively. The higher the salinity difference between the fed and the produced water, the more the osmotic pressure drop we need to overcome in order to produce treated water (positive flux).

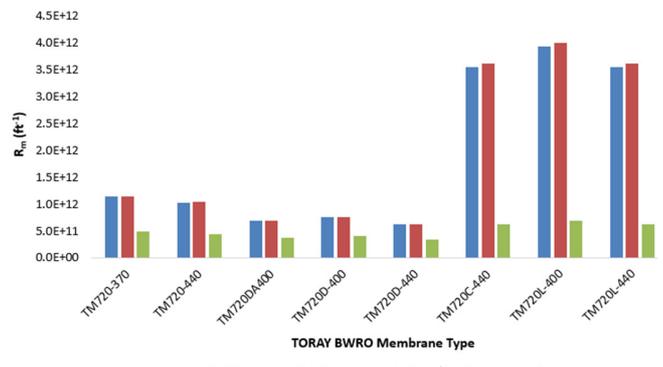


Fig. 3. Observed membrane resistance of various water sources in Toray BWRO membranes.

TABLE VIII. VAN'T HOFF CALCULATIONS FOR THE REQUIRED OSMOTIC PRESSURES

Water source	Concentration (mol/L)		Membrane removal (%)	Osmotic pressure (atm)			Δπ  (psi)
	TDS <sub>in</sub>	TDS <sub>out</sub>		π <sub>in</sub>	π <sub>out</sub>	Δπ	
Shedgum/Abqaiq groundwater	0.156	0.00031	99.8	3.81	0.01	3.80	55.80
Arabian Gulf	2.282	0.005	99.8	55.82	0.11	55.71	818.41
Red Sea	2.337	0.005	99.8	57.18	0.11	57.07	838.34

V. CONCLUSION

The application of the solution-diffusion model to the Abqaiq plant (500 RO plant) is initiated by using various parameters to calculate the osmotic pressure of Toray TM720D-400 with 8" membrane for Shedgum/Abqaiq groundwater treatment. For the same membrane, the osmotic pressure values are determined for the Arabian Gulf and the Red Sea waters to predict flux rates in other membranes for seawater situations. Low, standard, and high pressure BWRO Toray membranes performances have been compared to identify the optimal membrane for treating saline water from the three studied water sources at the Abqaiq 500 RO plant.

The assumption of having a membrane thickness that is similar to its spacer thickness may not seem very accurate. However, it is true that we should have a proportional relation between both thicknesses which suggests that our results are still valid. A linear relationship has been observed between the water flux and the applied pressure drops. It is proved that membrane flux decreases with the increase in membrane thickness at constant pressure drop. Modeling results endorse that BWRO Toray TM720D-440 with 8"membrane is the optimum membrane choice for the water treatment from the three water sources at Abqaiq 500 RO plant since it has the lowest membrane resistance and the highest overall water flux.

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