Empirical Modeling of Flow Behavior over Stepped Weirs Using Brink Depth for Sustainable Irrigation Applications

Very Dermawan
Department of Water Resources Engineering, Faculty of Engineering, Universitas Brawijaya, Indonesia
perydermа@ub.ac.id (corresponding author)

Edijatno
Department of Civil Engineering, Faculty of Civil Planning and Geo Engineering, Institut Teknologi Sepuluh Nopember, Indonesia
edijatno@yahoo.com

Runi Asmaranto
Department of Water Resources Engineering, Faculty of Engineering, Universitas Brawijaya, Indonesia
runi_asmaranto@ub.ac.id

Evi Nur Cahya
Department of Water Resources Engineering, Faculty of Engineering, Universitas Brawijaya, Indonesia
evi_nc@ub.ac.id

Nenny Roostrianawaty
Department of Water Resources Engineering, Faculty of Engineering, Universitas Brawijaya, Indonesia
nennyroos.nr@gmail.com

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ABSTRACT

Accurate and efficient discharge estimation is significant for optimizing irrigation system performance, particularly in resource-constrained regions such as Indonesia. Conventional discharge measurement techniques often rely on auxiliary equipment that is vulnerable to damage and requires extensive maintenance, especially in stepped-weir structures. This study introduces brink depth (y_b) as a practical and reliable parameter for predicting discharge and energy dissipation over inclined smooth and stepped weirs. Laboratory experiments were conducted with six slope angles (26.57°-72°), five step numbers (N = 2-32), and discharge rates ranging from 1.73 to 6.15 L/s. The results demonstrated a strong correlation between brink depth and discharge, enabling accurate prediction of energy dissipation. The maximum average relative energy loss reached 55.64% for the inclined smooth weir and 92.58% for the stepped weir. The proposed empirical equations yielded a Mean Absolute Percentage Error (MAPE) of 10.41% for the smooth weir and 12.96% for the stepped weir, confirming their robustness and applicability. This study provides a cost-effective, field-oriented methodology for discharge estimation and supports sustainable irrigation management.

Keywords-brink depth; stepped weir; discharge estimation; energy dissipation; irrigation; hydraulic modeling

I. INTRODUCTION

Accurate discharge estimation in open channel flow is essential for the effective operation of hydraulic structures such as stepped weirs, where complex flow regimes and geometric variability pose significant challenges. Among the available predictive parameters, brink depth (y_b), measured at the point of flow separation, has proven effective, particularly under free overfall conditions [1, 2]. However, discharge prediction for inclined and stepped weirs is more complex due to the transitions between nappe and skimming flow regimes [3, 4].

Previous studies have demonstrated that discharge coefficients are highly sensitive to crest geometry, highlighting the need for improved predictive models [5, 6]. Brink depth has been incorporated into empirical and machine-learning-based discharge equations to enhance prediction accuracy for non-standard weir geometries [7]. These approaches are particularly valuable for irrigation systems in low-resource settings, where durable and equipment-independent measurement methods are crucial [8]. Authors in [9] demonstrated that the discharge capacity of a compound structure involving rectangular elements and an underlying gate is significantly influenced by the structural dimensions, often delivering 2 to 10 times higher discharge. Furthermore, environmental factors, such as sediment transport and backwater effects, further complicate discharge estimation, emphasizing the need for comprehensive hydraulic modeling [10, 11].

Empirical discharge equations derived from physical models are significant in improving irrigation efficiency, particularly in rural regions experiencing climatic variability. Enhanced discharge prediction can support agricultural productivity in water-limited settings [12]. However, generic models often lack adaptability to diverse weir geometries. Data-driven techniques, including machine learning, have gained traction for predicting discharge coefficients with increased precision [13, 14]. Despite its potential, the use of brink depth for modeling energy dissipation remains underexplored, particularly in stepped weirs.

This issue is particularly important in Indonesia, where agriculture plays a central role in livelihoods and food security. Stepped weirs, such as those in Damarwulan, Kadalpang, Tukad Unda, Hantap Ciherang, Arca, and Mejagong, are widely implemented; yet, limited funding often results in inadequate discharge monitoring systems. In this context, brink depth provides a cost-effective and easily observable parameter for discharge estimation [15]. Although its applicability has been validated for sharp-crested and trapezoidal weirs [1], its use in stepped irrigation structures is limited, and the associated energy dissipation characteristics have not been thoroughly investigated [16, 17].

The present study addresses these gaps by investigating the use of brink depth as a predictor of discharge and energy loss in both stepped and inclined smooth weirs. Building upon evidence that brink depth models are less accurate under steep slopes or supercritical flow conditions [18], this study tests their applicability across various hydraulic scenarios. Laboratory experiments aimed to establish empirical equations for predicting both discharge and energy dissipation [19], offering a practical and low-cost alternative for improved irrigation management in Indonesia.

II. MATERIALS AND METHODS

Hydraulic modeling experiments were conducted in a laboratory open-channel flume to evaluate the flow behavior and energy dissipation associated with brink depth over both smooth and stepped weirs. The flume had a rectangular cross-section with a width of 0.30 m and a height of 1.00 m, allowing for direct visual observation of the turbulence development and hydraulic jump formation downstream. The channel length was adjusted for each slope configuration to prevent geometric distortion and ensure stable upstream and downstream flow conditions. A recirculating pump system supplied clean water, and a control valve was used to regulate the discharge. Discharge was measured using a 90° V-notch weir installed in the inlet line, ensuring repeatable and steady flows. To facilitate reliable observation of the conjugate depth downstream of the hydraulic jumps, a tailwater sill was calibrated based on standard momentum principles. This approach maintained consistent tailwater conditions across the experiments and minimized the bias caused by jump migration or incomplete formation. The dataset covers six slopes: 1:0.33 ($\theta = 72°$), 1:0.5 ($\theta = 63.44°$), 1:0.75 ($\theta = 53.13°$), 1:1 ($\theta = 45°$), 1:1.5 ($\theta = 33.69°$), and 1:2 ($\theta = 26.57°$). For stepped weirs, five step numbers (N = 2, 4, 8, 16, 32) were used to analyze the effect of step geometry across all slope conditions. The step height-to-length ratio (h/l) varied depending on the slope, with values of h/l = 3.08 for 1:0.33, h/l = 2.00 for 1:0.5, h/l = 1.33 for 1:0.75, h/l = 1.00 for 1:1, h/l = 0.67 for 1:1.5, and h/l = 0.50 for 1:2. Each combination of slope and step configuration was tested under five different discharges, ranging from 1.73 to 6.15 L/s, yielding a total of 150 experimental runs.

Flow depth measurements were taken using point gauges with an accuracy of ± 1 mm. Measurement points included the critical depth (y_c) at the weir crest, brink depth (y_b) at the downstream edge of the weir, the pre-jump depth (y_1), and the post-jump depth (y_2) at the downstream of the weir. The cross-sectional mean velocity was estimated using the continuity equation. The brink depth (y_b) was measured at the downstream edge of the weir crest, which represents the point of flow separation. The aeration effects were not corrected to maintain practical relevance to field applications. The flow regimes over the stepped weirs were classified based on the relative critical depth (y_c/h) as follows: nappe flow when (y_c/h) < 0.8 and skimming flow when (y_c/h) ≥ 0.8 [20].

Table I presents the experimental model configurations for smooth and stepped weirs, detailing the slope angle, step number, step dimensions, and critical depth ranges in each series.

Figure 1 illustrates typical inclined smooth and stepped weirs studied in the laboratory. Figure 2 presents photographs of the laboratory weir models with different configurations.

Energy dissipation was quantified using the relative energy loss formula (ratio of crest-to-toe energy loss ($\Delta E_1/E_0$)). The following equations were used:

$$y_c^3=\frac{Q^2}{B^2 g}=\frac{q^2}{g} \tag{1}$$

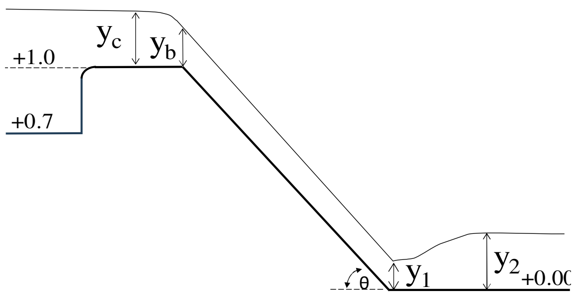
$$E=y+\frac{V^2}{2g} \tag{2}$$

$$MAPE=\frac{100\%}{n}\sum_{i=1}^{n}\left|\frac{y_i-\hat{y}_i}{y_i}\right| \tag{3}$$

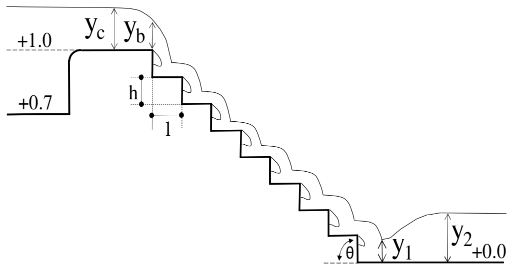
where y_c is the critical depth (m), Q is the discharge (m^3/s), V is the velocity (m/s), q is the discharge per unit width (m^2/s), B is the channel width (m), g is the gravitational acceleration (m/s^2), E is the flow energy (m), y is the flow depth (m), y_i is the expected flow depth (m), $\hat{y}_i$ is the observed flow depth (m), and MAPE is the Mean Absolute Percentage Error (%).

TABLE I. EXPERIMENTAL MODEL CONFIGURATIONS

Series	Weir	Step			Critical depth
		Number (N)	length l (cm)	Height h (cm)	y_c (cm)
I θ=72° (1:0.33)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	16.30	50.00	1.5; 2; 2.5; 3; 3.5
		4	8.10	25.00	1.5; 2; 2.5; 3; 3.5
		8	4.10	12.50	1.5; 2; 2.5; 3; 3.5
		16	2.00	6.30	1.5; 2; 2.5; 3; 3.5
		32	1.00	3.10	1.5; 2; 2.5; 3; 3.5
II θ=63.44° (1:0.5)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	25.00	50.00	1.5; 2; 2.5; 3; 3.5
		4	12.50	25.00	1.5; 2; 2.5; 3; 3.5
		8	6.30	12.50	1.5; 2; 2.5; 3; 3.5
		16	3.10	6.30	1.5; 2; 2.5; 3; 3.5
		32	1.60	3.10	1.5; 2; 2.5; 3; 3.5
III θ=53.13° (1:0.75)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	37.50	50.00	1.5; 2; 2.5; 3; 3.5
		4	18.80	25.00	1.5; 2; 2.5; 3; 3.5
		8	9.40	12.50	1.5; 2; 2.5; 3; 3.5
		16	4.70	6.30	1.5; 2; 2.5; 3; 3.5
		32	2.30	3.10	1.5; 2; 2.5; 3; 3.5
IV θ=45° (1:1)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	50.00	50.00	1.5; 2; 2.5; 3; 3.5
		4	25.00	25.00	1.5; 2; 2.5; 3; 3.5
		8	12.50	12.50	1.5; 2; 2.5; 3; 3.5
		16	6.30	6.30	1.5; 2; 2.5; 3; 3.5
		32	3.10	3.10	1.5; 2; 2.5; 3; 3.5
V θ=33.69° (1:1.5)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	75.00	50.00	1.5; 2; 2.5; 3; 3.5
		4	37.50	25.00	1.5; 2; 2.5; 3; 3.5
		8	18.80	12.50	1.5; 2; 2.5; 3; 3.5
		16	9.40	6.30	1.5; 2; 2.5; 3; 3.5
		32	4.70	3.10	1.5; 2; 2.5; 3; 3.5
VI θ=26.57° (1:2)	Smooth	-	-	-	1.5; 2; 2.5; 3; 3.5
	Stepped	2	100.00	50.00	1.5; 2; 2.5; 3; 3.5
		4	50.00	25.00	1.5; 2; 2.5; 3; 3.5
		8	25.00	12.50	1.5; 2; 2.5; 3; 3.5
		16	12.50	6.30	1.5; 2; 2.5; 3; 3.5
		32	6.30	3.10	1.5; 2; 2.5; 3; 3.5



(a)



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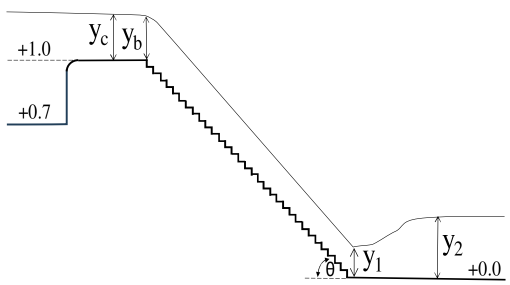

(c)

Fig. 1. Laboratory visualization of flow conditions over: (a) inclined smooth and stepped weirs, (b) nappe flow regime, and (c) skimming flow regime, highlighting differences in flow structure.

(a) 1:0.33 (θ=72^0), N=32
(b) 1:0.5 (θ=63.44^0), N=2
(c) 1:0.75 (θ=53.13^0), N=32
(d) 1:1 (θ=45^0), N=32
(e) 1:1.5 (θ=33.69^0), N=16
(f) 1:2 (θ=26.57^0), N=4

Fig. 2. Photographs of the weir model configurations under laboratory testing for various slope angles and step numbers, showing physical model variations.

To identify the dominant variables influencing energy dissipation in stepped weirs, dimensional analysis was applied to the variable set $f(y_1, y_2, y_c, E_0, \Delta E, V, q, l, h, g) = 0$. Using Buckingham's π-theorem with step height (h) and gravity (g) as repeating variables, the experimental interpretation focused on the dimensionless relationship among (y_c/h), (y_1/y_c), (y_c/y_1), and energy ratio [($\Delta E_1/y_c$), (E_1/y_c), (E_1/y_1)], supported directly by experimental data.

III. RESULTS AND DISCUSSION

The experimental results indicate that the flow behavior over inclined smooth and stepped weirs is primarily governed by the interaction of discharge intensity, slope angle, and step geometry. These parameters affect the formation of brink depth and the associated flow regimes. Among the 150 experimental runs, nappe flow predominated, whereas skimming flow occurred in 18 cases for the highest step number (N = 32) and $y_c \geq 2.5$ cm. The location of the critical flow on the smooth weir was observed at an average distance of approximately 3.7 y_c upstream from the weir edge.

A. Brink Depth-Based Discharge Estimation of the Weir

Brink depth (y_b) exhibited consistent sensitivity to variations in slope and step configurations, reflecting changes in aeration intensity, flow separation, and boundary-layer development along the chute. These results confirm that brink depth is a reliable hydraulic measurable parameter that can be used for discharge estimation under both smooth and stepped weir conditions. Table II presents the average value of the ratio y_b/y_c across all weirs, demonstrating the influence of step geometry on brink depth. The findings of this study substantiate the applicability of brink depth as a reliable parameter for estimating discharge (Q) in rectangular weir flows. A comparative analysis of experimental stepped-weir data revealed consistent patterns.

TABLE II. AVERAGE RATIO OF BRINK DEPTH TO CRITICAL DEPTH (y_b/y_c) ACROSS DIFFERENT SLOPES AND STEP NUMBERS

Series	Smooth weir	Stepped weir				
		N=2	N=4	N=8	N=16	N=32
I (1:0.33)	0.8251	0.7195	0.7842	0.7729	0.7029	0.7462
II (1:0.5)	0.8540	0.8393	0.8410	0.8370	0.8359	0.8230
III (1:0.75)	0.8360	0.8370	0.8193	0.8467	0.8600	0.7395
IV (1:1)	0.7148	0.6968	0.6860	0.6473	0.7365	0.6750
V (1:1.5)	0.8513	0.8613	0.8191	0.8176	0.8527	0.8490
VI (1:2)	0.8567	0.8553	0.8273	0.8367	0.7773	0.8073
Average	0.8230	0.8015	0.7961	0.7930	0.7942	0.7733
		0.7916				

If the average value of y_b/y_c = c, then y_c = (1/c) y_b, and according to Table II we get:

1. Inclined smooth weir: $y_c=1.2151\ y_b$ (4)
2. Stepped weir:
   a. All flow regimes : $y_c=1.2632\ y_b$ (5)
   b. Nappe flow: $y_c=1.2476\ y_b$ (6)
3. Skimming flow: $y_c=1.3363\ y_b$ (7)

The substitution of y_c with y_b in (1) and (4-7) allows the derivation of the following discharge formulation for rectangular weirs: $y_c^3=(Q^2/B^2g)$, which gives $Q=(By_c^{3/2}g^{1/2})$. Then, by utilizing the relationship between y_c and y_b obtained from the data analysis, the final discharge equations can be derived:

For an inclined smooth weir:

$$Q=1.3394\ B\ y_b^{3/2}g^{1/2} \quad (8)$$

For stepped weirs and all flow regimes:

$$Q=1.4197\ B\ y_b^{3/2}g^{1/2} \quad (9)$$

For a stepped weir and nappe flow regimes:

$$Q=1.3935\ B\ y_b^{3/2}g^{1/2} \quad (10)$$

For a stepped weir and skimming flow regimes:

$$Q=1.5447\ B\ y_b^{3/2}g^{1/2} \quad (11)$$

The proposed discharge formulas (8) and (9) demonstrated good agreement with the laboratory data, with average MAPE of 10.41%, 12.56%, 12.89%, 11.50%, 15.06%, and 12.80% across various slopes and flow regimes, as shown in Table III. These results confirm the reliability of brink depth as a discharge predictor without the need for upstream head measurements.

TABLE III. MAPE OF DISCHARGE ESTIMATION BASED ON BRINK DEPTH FROM (8)-(9)

Series	MAPE (%)					
	Smooth	N=2	N=4	N=8	N=16	N=32
I (1;0.33)	10.67	13.99	13.82	5.37	19.18	9.07
II (1:0.5)	7.44	9.23	12.34	11.54	15.65	11.31
III (1:0.75)	6.83	8.79	9.61	10.85	16.60	16.97
IV (1:1)	18.90	17.29	19.11	25.92	16.79	21.21
V (1:1.5)	8.71	13.57	8.17	6.56	11.93	12.78
VI (1:2)	9.88	12.49	14.25	8.76	10.23	5.43
Average	10.41	12.56	12.89	11.50	15.06	12.80
		12.96				

When averaged over the discharges and slopes, the smooth weir produces a higher mean y_b/y_c. This behavior is physically consistent with step-induced macro-roughness, which enhances turbulence and aeration and consequently reduces the brink depth relative to the critical control at the crest. The systematic mapping of y_b/y_c across six slopes and five step numbers provides a coherent basis for using brink depth as a practical hydraulic observable under both smooth and stepped conditions. The stepped and skimming flow coefficients are higher, indicating that for a given measured brink depth, the inferred critical depth, and thus discharge, must be larger, which aligns with stronger turbulence/aeration and modified velocity distributions near the brink in stepped and skimming flows.

B. Measurement Uncertainty and Sensitivity Analysis

The uncertainty of brink depth was considered by using a point gauge (±1 mm), which can be utilized to estimate the accuracy of the discharge equations. The discharge equations scale with y_b so that the relation $\delta Q/Q \approx (3/2)\ (\delta y_b/y_b)$ is found.

This indicates that discharge predictions are sensitive to errors at small brink depths and less so for those that occur at large depths. Under low-flow conditions, even small workarounds may result in larger uncertainties during the estimation of discharge. This is significant because the brink depth is an in-situ field measurement.

With point gauge accuracy of ±1 mm and using the first-order relative uncertainty $\delta Q/Q \approx (3/2)\ (\delta y_b/y_b)$, which corresponds to ~15% at y_b=1 cm, decreasing to ~3.75% at y_b=4 cm for δy_b=1 mm.

C. Applicability Range of the Developed Equations

The empirical equations presented in this study were derived from controlled measurements conducted in a 0.30 m wide rectangular flume. They are valid only for the laboratory conditions tested and for similar smooth weir configurations with stepped weir formats. However, the proposed equations should be applied with caution to field-scale structures with different geometric or boundary conditions than those used in this study.

The equations developed in this study are consistent with the general hydraulic form commonly used in brink-depth-based discharge estimation, in which discharge is proportional to $By_b^{3/2}g^{1/2}$. However, compared with previous studies on conventional sharp-crested weirs, the proposed equations were derived from inclined smooth and stepped weirs with more complex flow structures. This difference is important because the transition between nappe and skimming flows, together with the geometric influence of slope and step number, affects the empirical coefficients and contributes to higher prediction uncertainty than that typically reported for simpler weir configurations.

The present study extends the use of brink depth as a practical predictor for discharge estimation in non-standard weir geometries. Unlike previous studies that focused mainly on conventional crest shapes or isolated stepped configurations, this work considers a wider combination of slopes and step numbers, and therefore provides a broader experimental basis for evaluating the hydraulic behavior of inclined and stepped weirs.

D. Energy Dissipation on the Weir

Comparative analysis revealed that stepped weirs dissipated significantly more energy than smooth weirs. The stepped weir increased conjugate depths y_1 and y_2 and reduced the hydraulic jumps. The energy dissipation results further highlight the hydraulic advantage of stepped configurations. Crest-to-toe relative energy loss ($\Delta E_1/E_0$) averaged 55.64% for the smooth weir, whereas the stepped weirs achieved an overall mean dissipation of 92.13%. For a constant slope across all step numbers, the maximum dissipation occurred at 1:0.5 (θ = 63.44°), reaching an average of 93.92%, whereas for a constant step number across slopes, N = 32 produced the highest average dissipation (92.58%). The enhanced dissipation in stepped configurations is attributed to the combined losses from flow separation and reattachment, recirculation within step cavities, and intense turbulence generation along the chute. Table IV shows the average relative energy loss ($\Delta E_1/E_0$), demonstrating significant energy dissipation enhancements due to the stepped configurations.

The free-flow conditions do not always yield higher energy dissipation than skimming flow. Long weirs tend to dissipate more energy under skimming flow on flatter slopes, whereas short, steep weirs favor free-flow dissipation. In stepped weir, energy dissipation occurs primarily along the step surfaces due to flow disturbances. These disturbances substantially reduce energy, producing subcritical flow at the toe with high turbulence from water jets. The hydraulic jump downstream dissipates less energy than the step-induced turbulence.

Although many studies distinguish between nappe and skimming mechanisms, the crest-to-toe dissipation in this dataset is similarly high for both regimes (92.10%-92.20%). This indicates that within the tested range and using crest-to-toe accounting, geometric control (θ and N) dominates the dissipation response, whereas regime differences are partially confounded by the fact that skimming occurred only at N = 32.

TABLE IV. RELATIVE ENERGY LOSSES ($\Delta E_1/E_0$) FROM CREST TO TOE ACROSS ALL WEIRS

Slope	Smooth weir	Stepped weir					
		N=2	N=4	N=8	N=16	N=32	Average
1:0.33	15.89	91.51	91.88	91.60	91.42	93.02	91.89
1:0.50	68.59	92.45	94.97	93.66	94.89	93.64	93.92
1:0.75	56.63	92.12	91.69	91.58	91.82	92.44	91.93
1:1	52.34	91.89	91.89	91.61	92.45	93.14	92.19
1:1.5	72.05	90.76	91.24	90.36	91.42	91.38	91.03
1:2	68.35	91.28	91.79	90.82	93.24	91.83	91.79
Average	55.64	91.67	92.24	91.60	92.54	92.58	92.13

The obtained data showed that the most influential parameters to energy dissipation on the stepped weir can be expressed as $\Delta E_1/y_c = f(y_c/h)$, $(E_1/y_c) = f(y_1/y_c)$, and $(E_1/y_1) = f(y_c/y_1)$. Figure 3 illustrates the relationship between $\Delta E_1/y_c$ and y_c/h for the stepped weir at 1:0.5 (63.44°) across various step numbers. This configuration yielded maximum dissipation.

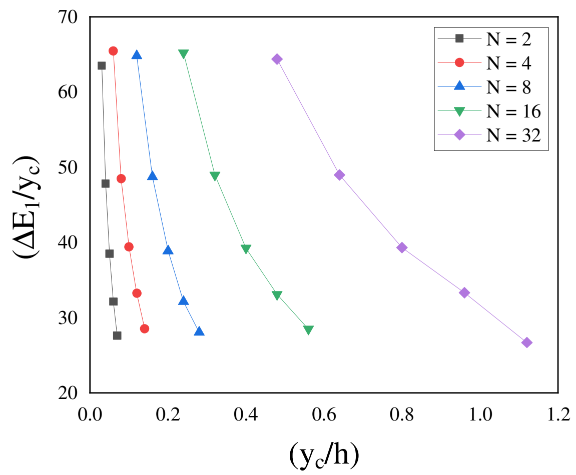

Fig. 3. Relationship between y_c/h and $\Delta E_1/y_c$ for 1:0.5 (63.44°) stepped weir for all step numbers.

Figure 4 presents the relationship between $\Delta E_1/y_c$ and y_c/h for N=32 across all slopes, experiencing the highest energy dissipation among others. Figures 5 and 6 display the relationship between y_1/y_c and E_1/y_c for the stepped weir across all flow regimes and nappe flow conditions based on dimensional analysis and polynomial fitting, respectively. The polynomial fit indicated strong correlation ($R^2 > 0.96$ and $R^2 > 0.99$).

Figures 7-9 illustrate how the interaction between the step geometry and flow regime governs the toe energy under skimming flow. Figure 7 presents the relationship between y_c/y_1 and toe energy E_1/y_1 under skimming flow across all slopes, exhibiting clear nonlinearity and several divergent data points and outliers correspond to the 63.44° stepped weir. This deviation is due to the shallow depth at the weir toe, which induces supercritical flow ($F_1>1$).

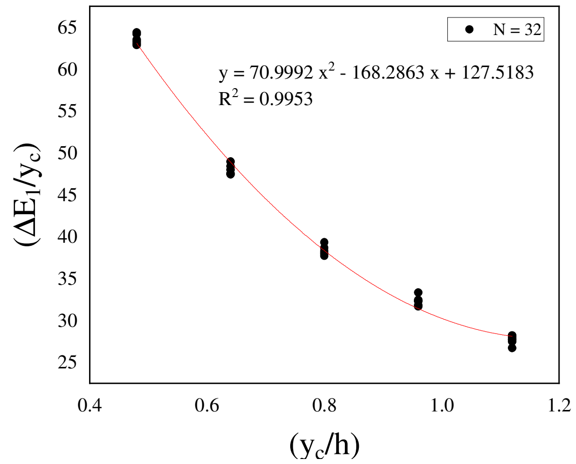

Fig. 4. Relationship between y_c/h and $\Delta E_1/y_c$ for stepped weir N=32 for all slopes.

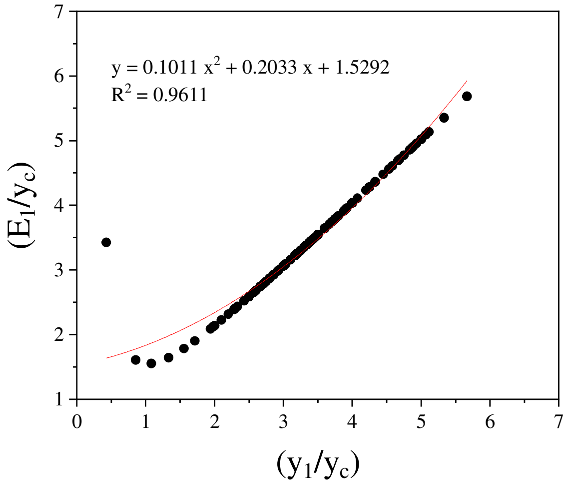

Fig. 5. Relationship between y_1/y_c and E_1/y_c for stepped weir across all flow regimes.

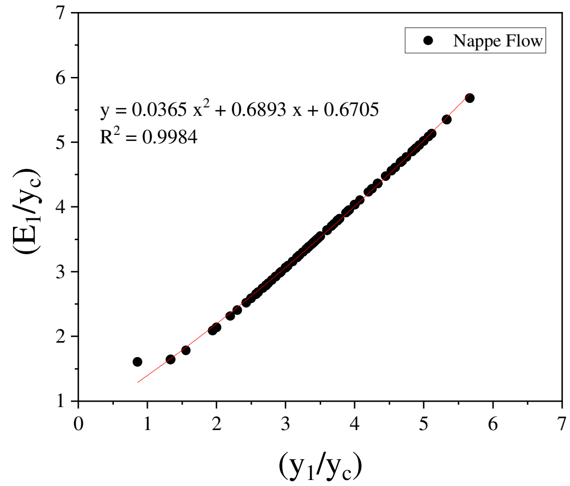

Fig. 6. Relationship between y_1/y_c and E_1/y_c for stepped weir under nappe flow condition across all slopes.

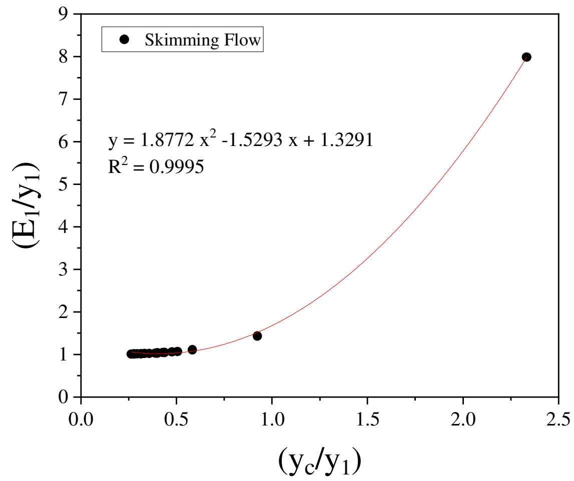

Fig. 7. Relationship between y_c/y_1 and E_1/y_1 for stepped weir under skimming flow across all slopes.

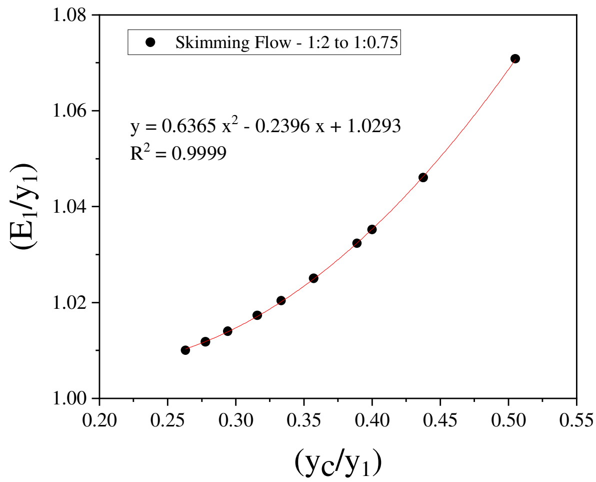

Fig. 8. Relationship between y_c/y_1 and E_1/y_1 for stepped weir 26.57° (1:2) to 53.13° (1:0.75) under skimming flow.

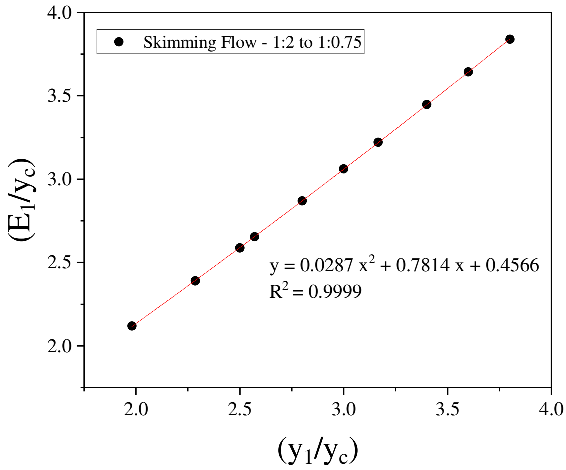

Fig. 9. Relationship between y_1/y_c and E_1/y_c for stepped weir 26.57° (1:2) to 53.13° (1:0.75) under skimming flow.

Figures 8 and 9 depict the polynomial fit of dimensionless parameter relationships for the stepped weir under skimming flow at slope angles <63.44° (1:0.5), indicating a strong correlation ($R^2 > 0.99$).

The equations for the energy at the toe of the stepped weir, downstream pre-jump, are:

All flow regimes:

$$\left(\frac{E_1}{y_c}\right)=0.101\left(\frac{y_1}{y_c}\right)^2+0.2033\left(\frac{y_1}{y_c}\right)+1.5292 \tag{12}$$

Nappe flow:

$$\left(\frac{E_1}{y_c}\right)=0.0365\left(\frac{y_1}{y_c}\right)^2+0.6893\left(\frac{y_1}{y_c}\right)+0.6705 \tag{13}$$

Skimming flow:

$$\left(\frac{E_1}{y_1}\right)=1.8772\left(\frac{y_c}{y_1}\right)^2-1.5293\left(\frac{y_c}{y_1}\right)+1.3291 \tag{14}$$

The energy at the toe of the stepped weir under skimming flow for slopes ranging from 1:2 (26.57°) to 1:0.75 (53.13°) can be approximated as:

$$\left(\frac{E_1}{y_1}\right)=0.6365\left(\frac{y_c}{y_1}\right)^2-0.2396\left(\frac{y_c}{y_1}\right)+1.0293 \tag{15}$$

$$\left(\frac{E_1}{y_c}\right)=0.0287\left(\frac{y_1}{y_c}\right)^2+0.7814\left(\frac{y_1}{y_c}\right)+0.4566 \tag{16}$$

Equations (15) and (16) have high accuracy, with MAPEs of 0.02% and 0.26%, respectively.

The obtained data on conjugate depth showed good agreement with the pre-jump and post-jump Froude numbers. The turbulence of the flow due to the arrangement of steps along the weir spine increases in the value of y_1, which is then due to the flow jet from the previous step and the occurrence of a hydraulic jump, so the value of y_2 becomes relatively higher. Thus, the conjugate depth y_2/y_1 is also higher than that of the smooth weir.

The relationship between conjugate depth (y_2/y_1) and pre-jump Froude number F_1 is shown in Figure 10, exhibiting a nonlinear increasing trend, in agreement with the established momentum-based formulation of hydraulic jump, indicating depth increase due to energy loss in stepped configurations. Furthermore, Figure 11 demonstrates that the normalized downstream depth y_2/y_c decreases with increasing post-jump Froude number F_2, supporting the notion that greater subcritical depth leads to lower flow velocity and energy. These results confirm that the empirical or simulated data accurately capture the physics of supercritical-to-subcritical transitions, energy dissipation, and depth-velocity interactions.

Brink depth is a reliable parameter for discharge estimation and energy dissipation analysis over stepped and inclined weirs. The experimental results of this study confirm that the discharge can be accurately predicted using the brink depth. Additionally, maximum energy dissipation occurred under skimming flow conditions on steep slopes and with a high number of steps (N). These findings align with broader trends in hydraulic engineering, where simplified yet robust empirical models are favored for flow analysis. Previous works have validated the use of brink depth across various geometries. For instance, authors in [1] developed a slope-independent brink depth-based formula for sharp-crested weirs with high predictive accuracy (MAPE = 1.71%), while in [2], the reliability of the End Depth Ratio (EDR) in semi-circular and triangular channels was emphasized.

The current study extends this approach to stepped configurations, providing new data on the nappe and skimming flow regimes. The adoption of end sills or curved steps was found to increase energy dissipation rates compared to traditional steps [21]. The brink depth also exhibits a strong correlation with the discharge coefficient (C_d). According to [19], variations in y_b affect C_d and improve discharge predictions, especially under high-flow conditions, where upstream head measurements become impractical [22]. The discharge coefficient values range from 0.86 to 0.93 [19] and from 0.3 to 0.9 [21], respectively.

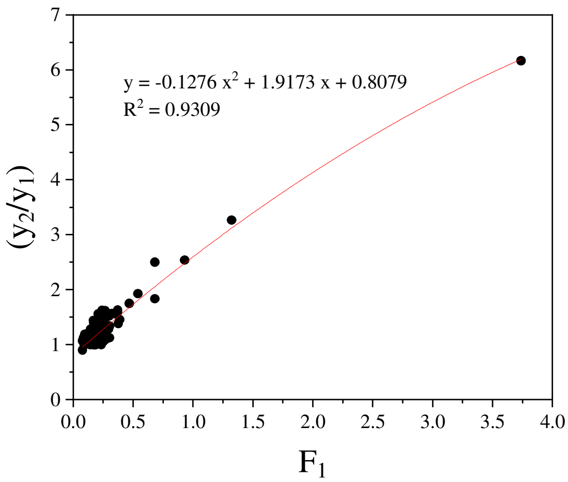

Fig. 10. Relationship between y_2/y_1 and F_1 for stepped weir across all slopes.

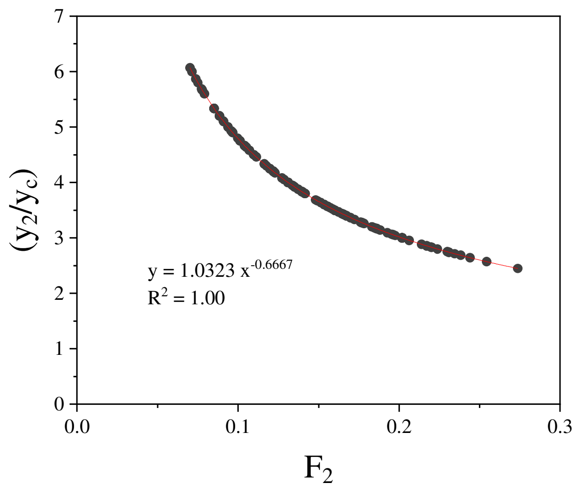

Fig. 11. Relationship between y_2/y_c and F_2 for stepped weir across all slopes.

Moreover, y_b models effectively dissipate energy by capturing the hydraulic response to geometric and flow variations [16, 17]. Recent validations have further reinforced the utility of brink-based models. Authors in [1] demonstrated low error margins across sharp-crested weirs, while numerical and hybrid algorithm studies confirmed the robustness of brink depth formulations across a wide range of weir configurations [23, 24]. These models offer high accuracy without increased computational complexity, supporting broader adoption in engineering applications [15, 2].

Slope angle (θ) significantly affects the accuracy of brink depth-based discharge estimation. Authors in [18] showed that C_d remains stable at moderate slopes but becomes variable at steeper inclinations. Similarly, authors in [26] demonstrated high model performance across varied slopes ($R^2 > 0.995$). Nappe flow, characterized by free-falling jets, yields higher dissipation (up to 90%) due to enhanced turbulence and aeration [3, 4].

Skimming flow, although more uniform and stable at high discharges, results in lower energy losses due to reduced mixing [27]. However, geometry plays a crucial role: step modifications can enhance dissipation by up to 15%, especially when promoting nappe flow or increasing turbulence intensity [28, 29]. Meanwhile, studies on stepped cylindrical weirs have shown that they achieve approximately 10% greater energy dissipation than traditional stepped weirs, with a maximum energy dissipation of 67.27% [30].

The higher crest-to-toe energy dissipation observed in stepped weirs can be explained by the hydraulic mechanisms induced by the stepped geometry. At each step edge, the flow undergoes repeated separation and reattachment, while the cavities between steps generate recirculating vortices. These vortices promote momentum exchange, turbulence production, and localized energy loss along the chute. In addition, the step configuration acts as macro-roughness, disrupting the flow and enhancing the conversion of mean flow energy into turbulent energy.

Under both nappe and skimming flow conditions, air entrainment and intense mixing further contribute to the dissipation process. As a result, the residual energy at the toe becomes substantially lower than that observed in smooth weirs. This explains why stepped weirs perform more effectively as energy dissipators and reduce the severity of the downstream hydraulic jump. The magnitude of dissipation, however, remains dependent on the slope angle, number of steps, and prevailing flow regime.

The integration of brink depth into discharge and energy loss models provides a practical and accurate approach for evaluating the hydraulic performance of stepped and inclined weirs. Although its predictive capability is well supported across flow regimes and geometries, slope sensitivity and flow regime behavior must be carefully considered to optimize model reliability and structural design.

The significance of empirical discharge estimation equations based on brink depth in irrigation is both profound and multifaceted. Accurate discharge estimation directly governs irrigation efficiency, influencing agricultural productivity and water conservation. The brink depth approach provides a robust framework for predicting discharge across diverse weir geometries, ensuring precise water delivery, which is essential for optimizing crop yields. Empirical equations incorporating brink depth have demonstrated their effectiveness in calculating flow rates under various irrigation scenarios and structural conditions, advancing canal design and hydrological modeling [31, 32].

E. Field Validation of the Proposed Equations

Field investigations and measurements were conducted on the Kadalpang Stepped weir in Betek, Malang City, which was constructed with river-stone masonry and has a height of 7 m and a width of 44 m (Figure 12). The five horizontal steps were designed with rounded corners with the following dimensions: the first step is 0.4 m high and 1.0 m long; the second, 0.6 m and 1.5 m; the third, 0.4 m and 2.5 m; the fourth, 0.6 m and 3.3 m; and the fifth, 5.0 m and 3.5 m. Approximately 5 m downstream of the last step, a large boulder creates a water cushion directly beneath it. Observational data were obtained through a series of 11 field measurement sessions. The recorded brink depth of the stepped weir ranged between 10 cm and 13.5 cm.

The validation results show that the proposed equations work particularly well for energy-related parameters. The relative errors for ΔE_1 and $\Delta E_1/E_0$ were only 0.79% and 1.21%, respectively, confirming that the developed relationships can reproduce the overall energy dissipation behavior of the stepped weir with high accuracy. The relative errors for y_1 and E_1 were 13.99% and 12.85%, respectively, which remain acceptable for field applications, given the irregularity of the prototype geometry. In contrast, larger discrepancies were noted for y_b, and especially y_2, with relative errors of 25.27% and 83.40%, respectively. The high error in y_2 is primarily due to the backwater effect induced by the downstream boulder, which increased the tailwater depth and altered the flow development compared with the laboratory model.

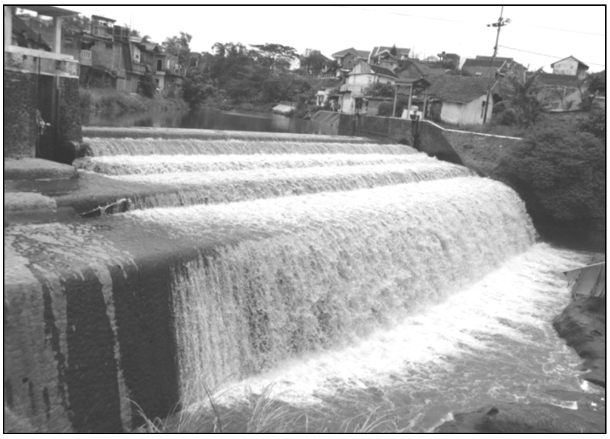

Fig. 12. Kadalpang stepped weir as a validation site.

Overall, the field validation confirmed that the proposed equations are more reliable for estimating global hydraulic performance, particularly energy dissipation, than for predicting local downstream depth conditions in prototype structures affected by irregular geometry and tailwater interference. This finding is hydraulically reasonable because energy dissipation reflects the integrated response of the stepped chute, whereas the downstream depth is strongly controlled by local boundary conditions.

The novelty and practical significance of this study lie in the unified experimental framework that connects (i) the systematic characterization of y_b/y_c across a broad slope-step matrix, (ii) direct brink-depth-based discharge estimation, and (iii) crest-to-toe energy dissipation quantification for identical inclined weir geometries. Unlike prior brink-depth studies, which largely focused on horizontal sharp-crested weirs or limited configurations, the present study provides a consistent, comparative dataset (150 runs) for inclined smooth and stepped profiles, delivering simple brink-based discharge equations and dimensionless toe-energy relations applicable to irrigation and grade-control structures, where upstream head measurements are unreliable.

IV. CONCLUSIONS

Brink-depth-based discharge equations provide an accurate and practical approach for both inclined smooth weirs ($Q=1.3394\ B\ y_b^{3/2} g^{1/2}$) and stepped weirs ($Q=1.4197\ B\ y_b^{3/2} g^{1/2}$). The proposed equations successfully predicted discharge with an average Mean Absolute Percentage Error (MAPE) of 10.41% for the smooth weir and 12.96% for the stepped weir, confirming that the brink depth is a useful hydraulic observable for discharge estimation without relying on upstream head measurement. In addition, the stepped weirs exhibited substantially higher crest-to-toe energy dissipation than the smooth weirs, with average values of 92.13% and 55.64%, respectively. This strong dissipation capability is attributed to the repeated flow separation and reattachment, cavity recirculation, turbulence generation, and macro-roughness effects along the stepped chute. Variations in slope angle and step number significantly influenced the flow regime behavior and energy dissipation, with moderate slopes (θ = 45°–50°) yielding the most stable discharge performance. These findings highlight the importance of geometric configuration in hydraulic modeling and support the application of brink-depth-based methods for sustainable irrigation infrastructure.

The proposed empirical relationships were derived from 150 controlled laboratory runs. The applicability of the proposed equations is currently limited to the hydraulic and geometric ranges investigated in the laboratory experiments and to field conditions with comparable stepped weir characteristics. Future research should focus on additional field validation, uncertainty assessment under operational conditions, and the influence of sediment transport, air entrainment, and unsteady flow on the predictive performance of brink depth-based hydraulic models.

DECLARATION OF COMPETING INTERESTS

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

AI USE AND DECLARATION OF GENERATIVE AI USE

During the preparation of this work, the authors used ChatGPT to assist in generating preliminary drafts of writing and refining the clarity of the text. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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