Multi-Criteria Evaluation of Brake Disc Materials Using BWM-TOPSIS Linear Programming

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ABSTRACT

The selection of the best all-around material for brake discs remains a challenging engineering problem due to the conflicting requirements for wear resistance, thermal stability, mechanical strength, and manufacturability. To address this multi-criteria challenge, this study proposes an integrated decision-making framework combining the index of Item-Objective Congruence (IOC), the Best-Worst Method (BWM), and the Technique for Order Preference by Similarity to Ideal Solution Linear Programming (TOPSIS-LP) approach. The IOC was employed to validate the relevance of six evaluation criteria based on input from five domain experts, while BWM derived weights for these criteria. TOPSIS-LP was then used to rank five material alternatives, including gray cast iron, steel alloy, Aluminum Metal Matrix Composite (Al-MMC), Carbon-Carbon/Carbon-Silicon Carbide Composite (C-C/SiC), and Ceramic-Based Composites (CMC), through linear optimization of the weighted decision matrix. The results indicated that the CMC achieved the highest performance with a relative closeness coefficient CC_i of 0.7629, followed by C-C/SiC and Al-MMC, while a four-scenario sensitivity analysis confirmed perfect rank stability (Spearman's rank correlation: $\rho_s = 1.00$) under ±20% weight variations. Overall, the findings indicate that advanced ceramic and composite materials outperform conventional metals in tribological and thermal performance, while the proposed IOC-BWM-TOPSIS-LP framework demonstrates strong robustness, interpretability, and applicability to complex engineering material selection problems.

Keywords-brake disc materials; Best-Worst Method (BWM); Technique for Order Preference by Similarity to Ideal Solution (TOPSIS); linear programming; Multi-Criteria Decision-Making (MCDM); material selection

I. INTRODUCTION

Materials play a central role in modern engineering design, as the selection of appropriate materials directly influences product performance, manufacturability, cost efficiency, and overall competitiveness [1]. Additionally, the suitability of each material depends on the physical, mechanical, thermal, and manufacturing-related attributes of the intended application [2, 3]. Moreover, with the rapid emergence of advanced materials offering superior performance compared with conventional alternatives [4], the decision-making process has become increasingly complex. Therefore, there is a need for a systematic and efficient methodology that can evaluate multiple attributes simultaneously and support reliable material selection for engineering applications.

In this study, we tackle the material selection decision process regarding automotive brake discs. Brake discs are often produced through casting and may undergo optional hardening treatments; thus, candidate materials must exhibit adequate wear resistance, thermal conductivity, thermal stability, hardness, toughness, machinability, and resistance to thermal cracking and corrosion. Furthermore, material costs must be evaluated to balance technical performance with economic feasibility [5, 6], as designers seek to reduce production expenses while maintaining safety and reliability [7].

For the material selection process, the assessment of suitability requires the evaluation of competing attributes, which, in the absence of a formal evaluation process, may result in inconsistent or subjective decisions. To address this issue, Multiple Criteria Decision-Making (MCDM) approaches have been widely employed to integrate expert judgment with quantitative performance data [8-10]. Among these, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is particularly effective due to its intuitive logic, computational efficiency, and ability to handle diverse criteria through normalization [11, 12].

Building on the foundation of classical TOPSIS, authors in [13] introduced the TOPSIS Linear Programming (TOPSIS-LP) model, which reformulates the ranking process as a linear optimization problem. This formulation reduces rounding and arithmetic errors, improves numerical stability, and provides a unified structure for multi-criteria evaluation. Recent studies have demonstrated the versatility of TOPSIS-LP across engineering and management contexts [14-16], while hybrid Best-Worst Method (BWM)-TOPSIS models have been applied in sustainability assessment, material selection, and Computer Numerical Control (CNC) machine evaluation [17-19]. However, applications of such hybrid frameworks in material selection for brake-disc systems remain limited, indicating a clear research gap.

Beyond TOPSIS-LP, numerous hybrid MCDM frameworks have been developed to enhance weighting reliability and decision robustness. For example, fuzzy BWM-TOPSIS accounts for uncertainty through fuzzy membership functions [20], whereas objective methods such as Criteria Importance Through Intercriteria Correlation (CRITIC)-TOPSIS derive criterion weights from contrast intensity and inter-criterion variability [21]. Other studies combine the Analytic Hierarchy Process (AHP) with linear or goal-programming formulations strengthen consistency under complex decision constraints [22]. Despite these advances, unresolved issues, including subjective fuzzification, the absence of systematic criterion validation, and the lack of a fully unified optimization structure, underscore the need for a more coherent and computationally stable approach to engineering material selection.

Among these, the BWM variant provides a coherent and effective approach that necessitates fewer pairwise comparisons than the AHP, while it is compatible with the TOPSIS-LP variant [23-25]

Accordingly, this article presents an integrated MCDM framework, utilizing the Index of Item-Objective Congruence (IOC) for criteria validation, the BWM for obtaining expert-derived weights, and the TOPSIS-LP model for ranking candidate materials.

This proposed framework aims to provide a systematic, transparent, and computationally reliable approach for identifying the most appropriate brake-disc material by combining quantitative performance metrics and expert evaluation.

II. METHODOLOGY

A. Research Framework

The research framework employed in this work is presented in Figure 1. The process began with a literature review to identify potential material alternatives and performance evaluation criteria. The identified criteria were subsequently validated by experts using IOC to ensure relevance and clarity, while BWM was then employed to derive criterion weights reflecting their relative importance. Finally, the TOPSIS-LP model was applied to evaluate and rank candidate materials based on their closeness to the ideal solution.

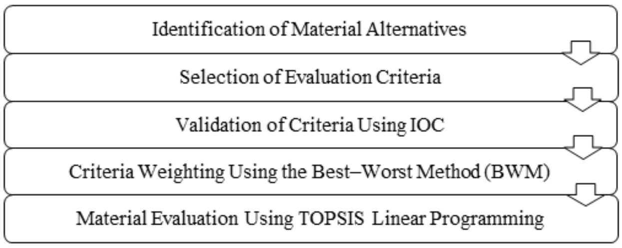

Fig. 1. The proposed methodological framework.

B. Identification of Material Alternatives

Recent studies employing MCDM-based material selection for automotive brake discs [5, 26-34] indicate a clear shift from conventional metallic materials toward advanced composite systems. Despite gray cast iron remaining the industrial benchmark due to its manufacturability and cost efficiency [34], steel alloys, Aluminum Metal Matrix Composites reinforced with Silicon Carbide particles (Al-MMC/Al-SiC), Carbon-Carbon/Carbon-Silicon Carbide Composite (C-C/SiC), and Ceramic-Based Composites (CMC) have gained increasing adoption in high-performance braking systems.

Based on these literature findings and expert consultation, five representative material families were selected for evaluation for this study: (1) gray cast iron, (2) steel alloys, (3) Al-MMC/Al-SiC, (4) C-C/SiC composites, and (5) CMC, SiC/Al.

To construct the decision matrix, material property values were collected from authoritative sources, including the MatWeb Materials Database, the ASM Handbook Series, and peer-reviewed experimental studies [28-34]. When multiple values were available, mid-range values were used to ensure comparability across materials. As a clarification, these data were adopted as representative parameters rather than absolute values measured under identical experimental conditions, and minor variations may arise due to differences in material grades, manufacturing routes, and testing orientations, particularly for composite materials. Therefore, while the data are appropriate for the comparative scope of the TOPSIS analysis, they may not fully reflect the actual mechanical or thermal behavior of production-grade brake discs.

For transparency, each material family was mapped to representative materials documented in the literature, as summarized in Table I.

TABLE I. MATERIAL ALTERNATIVES AND KEY CHARACTERISTICS

Alternative Material	Description / Key Characteristics	Representative Applications
A_1. Gray Cast Iron [5, 26, 29, 32, 34]	Conventional disc brake material with good damping capacity, high wear resistance, stable friction behavior, and low production cost.	Widely used in commercial automotive disc brakes.
A_2. Steel Alloys [26, 29-31, 34]	High mechanical strength and toughness; lower thermal conductivity; relatively heavier than other candidates.	Heavy-duty braking systems, trucks, and high-load applications.
A_3. Al-MMC/ Al-SiC [5, 25, 28, 29, 32]	Lightweight with high specific strength; improved heat dissipation; higher cost and increased brittleness compared to cast iron.	Lightweight vehicles, motorsport, and performance applications.
A_4. C-C/SiC [29, 32, 34]	Extremely high thermal stability, excellent wear resistance, and low density; very high manufacturing cost.	Aircraft braking systems, high-performance sports vehicles.
A_5. CMC, SiC/Al [5, 28, 32, 34]	High durability, superior thermal and oxidation resistance; comparatively expensive and specialized.	Premium automotive platforms and next-generation braking systems.

C. Selection of Evaluation Criteria

In parallel with the identification of material alternatives, an extensive literature survey was conducted to identify key criteria influencing brake disc performance in MCDM-based material selection studies [26, 29, 31-34].

This study extends the framework in [5], which identified compressive strength, coefficient of friction, wear resistance, thermal capacity, and specific gravity as key properties for brake disc design. To account for recent advances in composite and sustainable materials, additional criteria were incorporated based on recent studies [26-34]. In total, thirteen criteria were synthesized (Table II), covering mechanical, thermal, tribological, manufacturing, economic, and environmental aspects.

D. Validation of Criteria Using IOC

The preliminary evaluation criteria were validated using the index of IOC technique [35]. To ensure methodological rigor, a panel of five experts was selected, including specialists in materials engineering, manufacturing processes, thermo-fluids and Computational Fluid Dynamics (CFD), industrial engineering, and energy-environmental engineering. All experts have more than ten years of experience in fields directly related to brake disc materials and evaluation, ensuring the relevance and validity of the selected criteria. Each criterion was rated on a scale of -1 (irrelevant), 0 (uncertain), and +1 (relevant). The IOC value for each criterion was then computed using (1), where R_i represents the rating given by each expert and N is the total number of experts:

$$IOC_i = \frac{\sum R_i}{N} \quad (1)$$

Criteria with IOC scores equal to or greater than 0.50 were retained to ensure the validity and consistency of the evaluation framework with the research objectives.

TABLE II. EVALUATION CRITERIA SYNTHESIZED FROM LITERATURE

No.	Criterion	Description
1	Wear Resistance [5, 28, 32, 34]	Ability of the material to resist surface loss under frictional contact (inverse of wear rate).
2	Thermal Conductivity [5, 30, 34]	Capacity to dissipate heat generated during braking operation.
3	Strength-to-Weight Ratio [26, 29]	Balance between mechanical strength and mass efficiency.
4	Compressive Strength [5, 28, 34]	Resistance to compressive stress acting on the disc surface during braking.
5	Hardness [30, 32]	Resistance to plastic deformation and surface indentation.
6	High-Temperature Resistance [31, 34]	Ability to maintain structural integrity at elevated thermal conditions.
7	Coefficient of Friction [5, 33, 34]	Frictional interaction between disc and pad.
8	Manufacturability [26, 29]	Ease of forming, machining, and industrial processing for large-scale production.
9	Material Weight [5, 28, 30]	Density affecting rotational inertia and vehicle fuel efficiency.
10	Material Cost [29, 30]	Relative market price and availability for commercial use.
11	Recyclability [32, 34]	Potential for reuse and contribution to sustainable material cycles.
12	Service Life [33, 34]	Expected operational lifetime of the material under normal service conditions.
13	Environmental Performance [26, 34]	Stability under humidity, temperature fluctuation, and particulate exposure.

E. Criteria Weighting Using the BWM

The BWM was adopted to determine the relative importance (weights) of the evaluation criteria based on expert judgments. The method, originally proposed in [23], offers a structured and consistent framework for pairwise comparisons that require fewer comparisons and yield more reliable results compared to conventional approaches such as AHP.

The same five experts used in the IOC criteria validation participated in the BWM weighting stage. Initially, the experts identified the most important (best) and the least important (worst) criteria from the validated set. Subsequently, two comparison vectors were formed: i) the Best-to-Others (BO) vector a_{Bj}, and ii) the Others-to-Worst (OW) vector a_{jW}. Let $w = (w_1, w_2, \ldots, w_n)$ be the weight vector where $w_j \geq 0$ and $\sum w_j = 1$. To explicitly represent the optimization structure of BWM, the min-max problem can be formally expressed in symbolic linear programming form, in which the objective is to minimize the maximum deviation ξ between the preference ratios and the corresponding weight ratios. The optimal weights are obtained by solving the following min-max optimization problem:

$$\begin{aligned} \min_{w,\xi} \quad & \xi \\ s.t. \quad & \left| \frac{w_B}{w_j} - a_{Bj} \right| \leq \xi, \\ & \left| \frac{w_j}{w_W} - a_{jW} \right| \leq \xi, \\ & \sum_j w_j = 1, w_j \geq 0 \\ & j = 1,2,\dots,n \end{aligned} \tag{3}$$

where it can be simplified into linear programming form as:

$$\begin{aligned} \min_{w,\xi} \quad & \xi \\ \text{s.t.} \quad & | w_B - a_{Bj} w_j | \leq \xi, \\ & | w_j - a_{jW} w_W | \leq \xi, \\ & \sum_j w_j = 1, w_j \geq 0 \\ & j = 1,2,\dots,n \end{aligned} \tag{4}$$

where w_j denotes the weight of criterion j, w_B denotes the weight of the best criterion, while w_W denotes the weight of the worst criterion. To ensure the consistency of expert judgments, the Consistency Ratio (CR) is used, which is calculated as the ratio between the optimal deviation ξ^* and the maximum allowable deviation $\xi_{\max}$:

$$CR = \frac{\xi^*}{\xi_{\max}} \tag{5}$$

where ξ^* denotes the optimal value of the maximum deviation obtained from the BWM optimization model, and $\xi_{\max}$ represents the maximum allowable deviation corresponding to the adopted comparison scale. A CR value below 0.10 indicates satisfactory consistency among expert judgments.

Moreover, because more than one expert participated in the evaluation, their pairwise preference values (a_{Bj}, a_{jW}) were aggregated using the geometric mean method [24] before solving (3):

$$\begin{aligned} \bar{a}_{Bj} &= \left(\prod_{k=1}^{m} a_{Bj}^{(k)} \right)^{\frac{1}{m}}, \\ \bar{a}_{jW} &= \left(\prod_{k=1}^{m} a_{jW}^{(k)} \right)^{\frac{1}{m}} \end{aligned} \tag{6}$$

The aggregated values $(\bar{a}_{Bj}, \bar{a}_{jW})$ were then used in (4) to derive the final weights w^* for the TOPSIS analysis.

All BWM optimization models corresponding to (4) and (5) were implemented and solved using LINGO, ensuring stable convergence to the optimal deviation ξ^* and the associated weight vector w^*.

F. Material Evaluation Using TOPSIS-LP

To evaluate and rank the candidate brake disc materials, the TOPSIS-LP is employed, which integrates vector normalization and linear optimization into a single-step decision-making framework, enhancing mathematical rigor, computational efficiency, and robustness when handling multiple conflicting criteria [13]. This method can be partitioned into 4 stages:

1) Stage 1: Construction of the Decision Matrix

Let $A_i (i = 1,2,\dots,I)$ denote the set of candidate material alternatives and $C_j (j = 1,2,\dots,J)$ represent the evaluation criteria. The initial decision matrix $X = [x_{ij}]$ is constructed as:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1J} \\ x_{21} & x_{22} & \cdots & x_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ x_{I1} & x_{I2} & \cdots & x_{IJ} \end{bmatrix} \tag{7}$$

where x_{ij} denotes the performance value of alternative A_i with respect to criterion C_j.

2) Stage 2: Normalization of the Decision Matrix

Since the criteria are measured in different units and scales, vector normalization is applied to ensure comparability. The normalized decision matrix $Y = [y_{ij}]$ is defined as:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1J} \\ y_{21} & y_{22} & \cdots & y_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ y_{I1} & y_{I2} & \cdots & y_{IJ} \end{bmatrix} \tag{8}$$

For beneficial criteria (i.e., higher values are preferred), normalization is performed as:

$$y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{I} x_{ij}^2}} \tag{9}$$

whereas for cost criteria (i.e., lower values are preferred), inverse normalization is applied as:

$$y_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{I} x_{ij}^2}} \tag{10}$$

Through this process, all normalized criteria values are transformed into a benefit-oriented form, ensuring that higher y_{ij} values consistently represent superior performance prior to linear programming optimization.

3) Stage 3: Determination of Ideal Solutions

The positive ideal solution y_j^+ and negative ideal solution y_j^- for each criterion are defined as:

$$\begin{aligned} y_j^+ &= \max_i \{ y_{ij} \} \\ y_j^- &= \min_i \{ y_{ij} \} \\ j &= 1, 2, \dots J \end{aligned} \tag{11}$$

4) Stage 4: TOPSIS-LP Formulation

The metric used to quantify the proximity of each material alternative to the ideal solution was the relative closeness coefficient CC_i, with higher values indicating a more favorable material for brake disc applications.

To calculate the relative CC_i for each alternative, the TOPSIS-LP model introduces two non-negative decision variables, λ_i^- and λ_i^+ , representing the optimal weighting coefficients of the distances from the negative and positive ideal solutions, respectively. The objective function for each alternative A_i is formulated as:

$$\max\ CC_i = \lambda_i^- \sum_{j=1}^{J} \sqrt{w_j^2 \left(y_{ij} - y_j^-\right)^2} \tag{12}$$

subject to the normalization constraint:

$$\lambda_i^- \sum_{j=1}^{J} \sqrt{w_j^2 \left(y_{ij} - y_j^-\right)^2} + \\ + \lambda_i^+ \sum_{j=1}^{J} \sqrt{w_j^2 \left(y_j^+ - y_{ij}\right)^2} = 1 \tag{13}$$

and the non-negativity conditions:

$$\lambda_i^- \geq 0,\ \lambda_i^+ \geq 0,\ i = 1,2, \ldots, I \tag{14}$$

The TOPSIS-LP model defined in (12)-(14) was solved using the Microsoft Excel Solver add-in to obtain the relative CC_i through a single-step optimization process.

III. RESULTS AND DISCUSSION

A. Validation of Evaluation Criteria Using IOC

The results corresponding to the IOC method are summarized in Table III. Based on the average IOC values, six criteria, wear resistance (C_1), thermal conductivity (C_2), compressive strength (C_3), high-temperature resistance (C_4), coefficient of friction (C_5), and service life (C_6), achieved acceptable congruence levels (IOC ≥ 0.50).

TABLE III. IOC RESULTS FOR EVALUATION CRITERIA

Initial list of criteria	**IOC scores from experts**					**IOC Avg.**
	1	***2***	***3***	***4***	***5***	
Wear Resistance	1	1	1	1	0	0.80
Thermal Conductivity	-1	1	1	1	1	0.60
Strength-to-Weight Ratio	1	-1	1	1	0	0.40
Compressive Strength	1	1	1	1	0	0.80
Hardness	0	-1	1	0	0	0.00
High-Temperature Resistance	1	-1	1	1	1	0.60
Coefficient of Friction	1	1	1	1	1	1.00
Manufacturability	1	-1	0	0	-1	-0.20
Material Weight	1	-1	0	0	0	0.00
Material Cost	0	-1	0	1	-1	-0.20
Recyclability	0	-1	0	-1	-1	-0.60
Service Life	1	1	1	1	-1	0.60
Environmental Performance	0	-1	1	1	1	0.40

B. Criteria Weighting Using the BWM

Following the validation of evaluation criteria, BWM was applied to determine the relative importance of the six validated criteria. As indicated in Table IV, the five domain experts provided ratings on a scale of 1 to 9 to express the relative importance of each criterion, where higher values indicate stronger preference.

The aggregated scores shown in Table IV were obtained by summing the individual ratings assigned by the experts for each criterion. Based on these aggregated scores, criterion C_1 was identified as the most preferred (best) criterion, whereas criterion C_6 was identified as the least preferred (worst) criterion.

TABLE IV. PRELIMINARY EXPERTS' RATINGS FOR IDENTIFYING THE BEST AND WORST CRITERIA

Experts selected the best and worst criteria	**Criteria Rating Scale (1-9)**					
	C₁	***C₂***	***C₃***	***C₄***	***C₅***	***C₆***
Expert 1	9 / 6	7 / 5	6 / 4	7 / 5	5 / 3	1 / 1
Expert 2	5 / 2	4 / 1	6 / 3	8 / 5	7 / 4	9 / 6
Expert 3	9 / 7	6 / 4	4 / 2	8 / 6	7 / 5	5 / 1
Expert 4	7 / 3	9 / 5	6 / 2	9 / 5	8 / 4	5 / 1
Expert 5	8 / 5	9 / 6	7 / 4	5 / 2	6 / 3	4 / 1
Total score	38 / 23	35 / 21	29 / 15	37 / 23	33 / 19	24 / 10

Note: The first and second values indicate the preliminary Best and Worst ratings, respectively.

Afterwards, experts compared the best criterion against all other criteria (BO) and all other criteria against the worst criterion (OW), using the same 1-9 preference scale. The resulting pairwise comparison matrices based on individual expert opinions are presented in Table V.

TABLE V. PAIRWISE COMPARISON MATRIX BASED ON THE OPINION OF FIVE EXPERTS

Experts' Pairwise Comparisons	**BO (Best: C_1), OW (Worst: C_6)**					
	C₁	***C₂***	***C₃***	***C₄***	***C₅***	***C₆***
Expert 1	1 / 9	7 / 7	5 / 5	7 / 7	1 / 8	9 / 1
Expert 2	1 / 9	7 / 3	4 / 5	3 / 7	4 / 5	9 / 1
Expert 3	1 / 9	4 / 7	6 / 6	3 / 8	2 / 9	8 / 1
Expert 4	1 / 9	1 / 9	3 / 7	1 / 9	1 / 9	9 / 1
Expert 5	1 / 9	3 / 5	5 / 3	2 / 7	4 / 4	9 / 1

For the best criterion (C_1), the BO value is equal to 1 when compared with itself.

Since multiple experts participated in the evaluation, their individual BO and OW comparison values were aggregated using the geometric mean method. The resulting aggregated BO and OW vectors are reported in Table VI and serve as the input parameters for the BWM optimization model.

TABLE VI. RESULTS DATA FROM PAIRWISE COMPARISON BY THE GEOMETRIC MEAN

Criteria	***C₁***	***C₂***	***C₃***	***C₄***	***C₅***	***C₆***
BO (Best: C_1)	1	3.6	4.5	2.6	2	8.8
OW (Worst: C_6)	9	5.8	5.0	7.6	6.6	1

By solving the BWM optimization model, the optimal weights for each criterion were obtained and are reported in Table VII, showing that C_1 holds the highest importance (0.3611), followed by C_5 (0.2190) and C_4 (0.1685). Additionally, the calculated CR (0.0769) is below the acceptable threshold of 0.1, confirming the reliability of the expert judgments regarding the reported weights.

TABLE VII. CRITERIA WEIGHTS OBTAINED FROM BWM

Criteria	***C₁***	***C₂***	***C₃***	***C₄***	***C₅***	***C₆***
Weights	0.3611	0.1217	0.0974	0.1685	0.2190	0.0323

C. Material Ranking Using TOPSIS-LP

The five candidate materials were then evaluated following the TOPSIS-LP methodology. The raw decision matrix is presented in Table VIII, and the normalized results along with the CC_i are shown in Table IX. CMC achieved the highest CC_i (0.7629), followed by C-C/SiC (0.5517) and Al-MMC (0.4429). Accordingly, the final ranking of the materials is $A_5 > A_4 > A_3 > A_2 > A_1$, which is consistent with previous studies [5, 26].

TABLE VIII. DECISION MATRIX FOR ALTERNATIVE MATERIALS

Criterion	Unit	Direction	A_1: Gray Cast Iron	A_2: Steel Alloy	A_3: Al-MMC/ Al-SiC	A_4: C-C/SiC	A_5: CMC, SiC/Al
C_1:	10^6 mm³/N·m	Benefit	0.42	0.5	0.87	1.2	1.5
C_2:	W/m·K	Benefit	46	27	170	35	120
C_3:	MPa	Benefit	1100	1500	400	200	500
C_4:	°C	Benefit	500	600	400	900	1100
C_5:	–	Benefit	0.4	0.45	0.38	0.42	0.35
C_6:	10^3 km	Benefit	60	80	100	150	200

Based on the normalized decision matrix reported in Table IX, a radar chart was constructed to visualize the comparative performance of the five candidate materials across the six validated criteria, as shown in Figure 2. CMC (A_5) exhibits superior performance in wear resistance (C_1), high-temperature resistance (C_4), and service life (C_6), followed by C/C-SiC (A_4). On the other hand, Al-MMC (A_3) shows relatively higher performance in thermal conductivity (C_2), while the metallic alternatives (A_1, A_2) display more moderate and balanced profiles.

TABLE IX. NORMALIZED DECISION MATRIX AND CC_i

	C_1	C_2	C_3	C_4	C_5	C_6	CC_i	Rank
A_1	0.1903	0.2114	0.5563	0.2993	0.4456	0.2089	0.1935	5
A_2	0.2265	0.1241	0.7586	0.3592	0.5013	0.2785	0.2772	4
A_3	0.3941	0.7811	0.2023	0.2395	0.4233	0.3482	0.4429	3
A_4	0.5436	0.1608	0.1011	0.5388	0.4679	0.5222	0.5517	2
A_5	0.6795	0.5514	0.2529	0.6586	0.3899	0.6963	0.7629	1

D. Sensitivity Analysis

To examine the robustness of the proposed IOC-BWM-TOPSIS-LP framework, a sensitivity analysis was performed by systematically perturbing the criterion weights derived from the BWM process. Four scenarios (S_1-S_4) were generated by proportionally increasing the weights of the top-ranked criteria (C_1, C_4, and C_5) by 5%, 10%, 15%, and 20%, followed by normalization to maintain the total sum of one.

As summarized in Table X, the material ranking order remained unchanged across all perturbation scenarios, confirming the high stability of the proposed decision model. The CMC (A_5) consistently ranked first, followed by C-C/SiC (A_4) and Al-MMC (A_3), while steel alloy (A_2) and gray cast iron (A_1) remained the least preferred alternatives. The mean variation of the closeness coefficients ($|\Delta CC_i|$) increased slightly from 0.0063 at S_1 to 0.0229 at S_4, indicating minimal numerical fluctuation. Moreover, the Spearman's rank correlation (ρ_s) between the baseline and each scenario equaled 1.00, implying perfect correlation and no rank reversal.

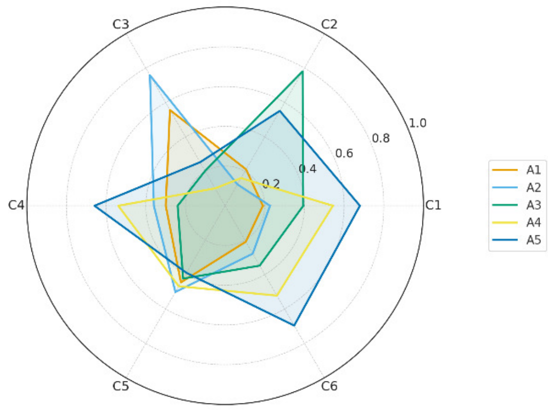

Fig. 2. Radar chart of normalized criteria performance.

TABLE X. VARIATION OF CC_i AND RANK ACROSS SCENARIOS S_1-S_4

Material Alternative	Baseline CC_i	Rank	S_1 (+5%) CC_i	Rank	S_2 (+10%) CC_i	Rank	S_3 (+15%) CC_i	Rank	S_4 (+20%) CC_i	Rank
A_1	0.1935	5	0.1878	5	0.1824	5	0.1775	5	0.1728	5
A_2	0.2772	4	0.2715	4	0.2661	4	0.2611	4	0.2564	4
A_3	0.4429	3	0.4384	3	0.4342	3	0.4304	3	0.4268	3
A_4	0.5517	2	0.5609	2	0.5694	2	0.5772	2	0.5846	2
A_5	0.7629	1	0.7695	1	0.7757	1	0.7814	1	0.7867	1
Mean ΔCC_i		—	0.0063	—	0.0123	—	0.0177	—	0.0229	—
ρ_s (B-S)		—	1	—	1	—	1	—	1	—

ρ_s (B-S): Spearman's rank correlation coefficient between baseline and scenario results.

IV. CONCLUSION

This study established a systematic Multi-Criteria Decision-Making (MCDM) framework to address the growing complexity of selecting brake disc materials. The proposed framework integrated Item-Objective Congruence (IOC), Best-Worst Method (BWM), and Technique for Order Preference by Similarity to Ideal Solution Linear Programming (TOPSIS-LP) for consistent criterion validation, weighting, and ranking of the materials examined.

Among the criteria examined, wear resistance, coefficient of friction, and high-temperature resistance were the dominant contributors. The results of our analysis also indicated that Ceramic-Based Composites (CMC) achieved the highest closeness coefficient (CC_i) of 0.7629, followed by Carbon-Carbon/Carbon-Silicon Carbide Composite (C-C/SiC) (0.5517) and Aluminum Metal Matrix Composites reinforced with Silicon Carbide particles (Al-MMC/Al-SiC) (0.4429).

Future extensions may incorporate fuzzy or interval BWM, entropy-based weighting methods, or fuzzy/interval TOPSIS-LP variants to enhance modeling flexibility and better capture uncertainty in expert judgments. Such developments would further strengthen the applicability and scalability of the proposed framework for high-dimensional and uncertainty-driven engineering problems.

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