A Similarity Measures-Based TOPSIS Method for Neutrosophic Hypersoft Set with Application in Crop Production

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Received: 13 October 2025 | Revised: 6 November 2025 and 24 November 2025 | Accepted: 26 November 2025

ABSTRACT

Neutrosophic sets and hypersoft sets are both fuzzy set extensions that deal with various aspects of uncertainty and missing data. Combining these two frameworks within Multi-Criteria Decision-Making (MCDM) allows choosing an optimal solution from a wide range of scenarios and alternatives. TOPSIS is a significant practical method for evaluating and selecting numerous possibilities, which ranks preferences according to how closely they resemble the ideal solution. This study used similarity and distance measures for NHSS and aggregate NHSS decision matrices by employing aggregation operators. The proposed NHSS-TOPSIS technique was used to assess the fertility of the soil for the production of specific crops. Multiple criteria were considered to make decisions regarding crop production based on soil conditions and other factors. This work can be further broadened to various existing hybrids of hypersoft sets, such as Intuitionistic Fuzzy Hypersoft Sets (IFHSS), Pythagorean Fuzzy Hypersoft Sets (PFHSS), Bipolar fuzzy hypersoft sets, Pythagorean Fuzzy Hypersoft Matrices (PFHSM), and neutrosophic hybrids.

Keywords-Multi-Criteria Decision-Making (MCDM); distance; similarity; Neutrosophic Hypersoft Set (NHSS); Neutrosophic Hypersoft Matrices (NHSM); TOPSIS

I. INTRODUCTION

In many real-world scenarios, decision-makers face the challenge of providing precise numerical information for their decisions. This is generally due to the inherent uncertainties, ambiguities, or subjective nature of the problem. MCDM approaches aim to deal with such scenarios by allowing decision-makers to use qualitative or ordinal evaluations rather than precise numerical values. Because of this adaptability, MCDM is useful in circumstances where precise quantification is difficult or impractical, considering numerous criteria at the same time to select a preferred alternative from various possibilities. This allows for a more thorough examination of the alternatives, resulting in well-informed selections. The idea of Fuzzy Sets (FS) was proposed in the 1960s [1] to address the constraints of classical set theory while addressing imprecision and uncertainty in the data to fill the gap. FS offers a framework for describing and manipulating notions with ambiguous or ill-defined boundaries. Following that, several modifications have been introduced, such as Intuitionistic Fuzzy Set (IFS) [2], type-2 FS [3], and Pythagorean Fuzzy Set (PFS) [4].

The Neutrosophic Set (NS) was introduced in 1998 [5] as a generalization of the aforementioned ideas, discussing truthiness, indeterminacy, and falsity. Following this, a Single-Valued Neutrosophic Set (SVNS) was introduced and implemented to resolve real-life challenges [6]. Soft Sets (SS), proposed in 1999, function as a revolutionary new mathematical technique to deal with uncertain challenges [7]. SS are termed a parameterized family of subsets of the universal set, where each member is considered a set of approximate SS elements [7]. In addition, SS aggregation operators were discussed, which were redefined further in [8], along with proposing a decision-making algorithm that uses different values of objects. In [9], the theoretical method of Fuzzy Soft Sets (FSS) was proposed, along with an algorithm to deal with challenges relevant to real life. SS theory has been combined with NS theory and extended in various ways. In [10], Interval-Valued Neutrosophic Set (IVNS) and SVNS were proposed. In [11], a Simplified Neutrosophic Set (SNS) was presented as a solution to problems, including a collection of a specific number of traits. In [12], a Neutrosophic Soft Set (NSS) was proposed. In [13], an extension of the TOPSIS technique for Multi-Attribute Decision-Making (MADM) was based on linguistic numbers. In [14], a normalized weighted mean operator was described for a simplified NS and applied to MCDM problems. In [15], the TODIM method was presented and applied with a rough NS. In [16], the multi-valued NS was introduced. Many researchers have worked on aggregation operators that depend on different formulations. In [17], Cubic M-polar fuzzy with T-conorm and T-norm for Dombi's was suggested.

Over the years, many SVNS models, including correlation coefficients, inclusion measures, similarity measures, entropy measures, and distance measures, have been proposed. In [18], many similarity measures for NSs were investigated. MCDM has been combined with a correlation coefficient [19] and an aggregation operator [20] for SVNS and SNS, and their vector similarity measures were discussed in [21]. In [22], some new SMs were developed. In [23-32], clustering methods for SVNS and data normalization methods were proposed. In [33], the limitations and solutions of the RWAEC methods were discussed. In [34], the Root Assessment Method (RAM) was proposed. In [35], particle applications and comparative analysis of MCDM methods were presented. In [36], Dombi aggregation operators were applied in SWMSS. In [37], a model for hydrogen gas production in stand-alone wind farms was proposed, while in [38], fuzzy models were applied in biological sciences. In [39], an approach was proposed to extend the SS to the Hypersoft Set (HSS) for dealing with multi-objective and multi-attribute problems in more uncertain circumstances. This approach has been extended into the Fuzzy Hypersoft Set (FHSS) [40] and the Multi-Fuzzy Hypersoft Set [41]. In [42], NHSS was combined with aggregate operators. NHSS mapping has been used to predict hepatitis [43] and suggest treatment for infectious disease diagnosis [44]. The study in [45] discussed how NHSS applications depend on decision-making. Similarity and distance measures have been proposed for NHSS by implementing max-min operators [46] and trigonometric similarity measures [47]. In [48], similarity measures (cosine and cotangent) were discussed for IFHSS, with implementation in the MADM problem. Table I shows some of the similarity measure formulas in the NHSS environment.

MCDM is a decision-making approach that entails analyzing and selecting options based on a set of criteria or objectives. It recognizes that judgments are typically complicated and comprise several factors that cannot be addressed completely by a single criterion. In traditional decision-making techniques, a single criterion is frequently used to evaluate possibilities. For example, when selecting a vendor, the most important factor may be price, and the vendor with the lowest price is chosen. However, MCDM recognizes that decisions might entail a variety of elements such as environmental impact, quality, risk, time, cost, and so on. MCDM techniques aim to form a systematic framework for assessing and contrasting alternatives based on these various criteria. These methodologies allow decision-makers to evaluate the pros and cons of various criteria. MCDM approaches that are regularly employed include VIKOR [49], PROMETHEE [50], and TOPSIS [51]. Several MCDM techniques, such as ARAS, MARCOS, VIKOR, RAFSI, MAIRCA, and AROMAN, have been widely applied in various decision-making environments. Each of these methods has its own computational structure and theoretical foundation.

In this study, the TOPSIS model was selected because it offers a simple, efficient, and intuitive mechanism for ranking alternatives based on their closeness to the negative ideal solution and distance from the positive ideal solution. Unlike VIKOR and MARCOS, which require additional parameters or compromise factors, TOPSIS depends on distance measures, making it computationally straightforward, especially when compared to RAFSI, ARAS, AROMAN, and MAIRCA, which involve more complex aggregation and normalization procedures.

TOPSIS [52] is one of the techniques that provide adequate solutions. The underlying idea behind TOPSIS is that the optimum solution is the one that simultaneously has the largest distance from the Negative Ideal Solution (NIS) and the lowest distance from the Positive Ideal Solution (PIS). TOPSIS is a prominent tool for handling MCDM. In [53], a comprehensive overview of TOPSIS applications in several sectors was presented. The selection of the optimal solution becomes tough in some real-world circumstances due to attribute dependence and subsequent bifurcation. To handle such cases, fuzzy TOPSIS, generalized fuzzy TOPSIS, and neutrosophic TOPSIS all failed. To deal with such circumstances, the NHSS-TOPSIS technique was presented in [54] using the Hamming distance and a similarity-based formula. This study applies the TOPSIS technique for NHSS by using distance and similarity measure-based formulas on NHSS with an application in the production of crops, and compares it with some existing approaches.

TABLE I. SIMILARITY MEASURES IN THE NHSS ENVIRONMENT

Study	Similarity Measure
[22]	$P^2(l.m)=\frac{\sum_{j=1}^{n}(\min(\Xi_{t_l}(\Theta_j),\Xi_{t_m}(\Theta_j))+\min\left(\Xi_{i_l}(\Theta_j),\Xi_{i_m}(\Theta_j)\right)+\min(\Xi_{f_l}(\Theta_j),\Xi_{f_m}(\Theta_j))}{\sum_{j=1}^{n}(\max(\Xi_{t_l}(\Theta_j),\Xi_{t_m}(\Theta_j))+\max\left(\Xi_{i_l}(\Theta_j),\Xi_{i_m}(\Theta_j)\right)+\max(\Xi_{f_l}(\Theta_j),\Xi_{f_m}(\Theta_j))}$
[23]	$Y^1(l,m)=1-\frac{1}{3}\sum_{j=1}^{n}w_j\left[\left\lvert\Xi_{t_l}(\theta_j)-\Xi_{t_m}(\theta_j)\right\rvert^p+\left\lvert\Xi_{i_l}(\theta_j)-\Xi_{i_m}(\theta_j)\right\rvert^p+\left\lvert\Xi_{f_l}(\theta_j),\Xi_{f_m}(\theta_j)\right\rvert^p\right]^{\frac{1}{3}}$
[24]	$JG_1(l,m)=1-\frac{1}{3\lvert X\rvert}\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert+\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert+\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)$ $JG_2(l,m)=1-\frac{1}{3\lvert X\rvert}\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert-\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert-\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)$ $JG_3(l,m)=1-\frac{1}{3\lvert X\rvert}\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert\in\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert\in\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)$ $JG_4(\hat{l},\hat{m})=\frac{1}{\lvert X\rvert}\sum_{\theta\in X}\frac{1-\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert\in\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert\in\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)}{1+\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert\in\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert\in\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)}$ $JG_5(l,m)=\frac{\sum_{\theta\in X}\left(1-\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert\in\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert\in\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)\right)}{\sum_{\theta\in X}\left(1+\left(\lvert\Xi^2_{t_l}(\theta)-\Xi^2_{t_m}(\theta)\rvert\in\lvert\Xi^2_{i_l}(\theta)-\Xi^2_{i_m}(\theta)\rvert\in\lvert\Xi^2_{f_l}(\theta)-\Xi^2_{f_m}(\theta)\rvert\right)\right)}$ $JG_6(l,m)=\alpha\frac{\sum_{\theta\in X}(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert}+\beta\frac{\sum_{\theta\in X}(\lvert\Xi^2_{i_l}(\theta)\wedge\Xi^2_{i_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{i_l}(\theta)\vee\Xi^2_{i_m}(\theta)\rvert}+\gamma\frac{\sum_{\theta\in X}(\lvert\Xi^2_{f_l}(\theta)\wedge\Xi^2_{f_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{f_l}(\theta)\vee\Xi^2_{f_m}(\theta)\rvert}$ $JG_7(l,m)=\frac{\alpha}{\lvert X\rvert}\frac{\sum_{\theta\in X}(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert}+\frac{\beta}{\lvert X\rvert}\frac{\sum_{\theta\in X}(\lvert\Xi^2_{i_l}(\theta)\wedge\Xi^2_{i_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{i_l}(\theta)\vee\Xi^2_{i_m}(\theta)\rvert}+\frac{\gamma}{\lvert X\rvert}\frac{\sum_{\theta\in X}(\lvert\Xi^2_{f_l}(\theta)\wedge\Xi^2_{f_m}(\theta)\rvert}{\sum_{\theta\in X}(\lvert\Xi^2_{f_l}(\theta)\vee\Xi^2_{f_m}(\theta)\rvert}$ $JG_8(l,m)=\frac{1}{\lvert x\rvert}\sum_{\theta\in X}\frac{\left(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert+\lvert\Xi^2_{i_l}(\theta)\wedge\Xi^2_{i_m}(\theta)\rvert+\lvert\Xi^2_{f_l}(\theta)\wedge\Xi^2_{f_m}(\theta)\rvert\right)}{\left(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert+\lvert\Xi^2_{i_l}(\theta)\vee\Xi^2_{i_m}(\theta)\rvert+\lvert\Xi^2_{f_l}(\theta)\vee\Xi^2_{f_m}(\theta)\rvert\right)}$ $JG_9(l,m)=\frac{\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert+\lvert\Xi^2_{i_l}(\theta)\wedge\Xi^2_{i_m}(\theta)\rvert+\lvert\Xi^2_{f_l}(\theta)\wedge\Xi^2_{f_m}(\theta)\rvert\right)}{\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert+\lvert\Xi^2_{i_l}(\theta)\vee\Xi^2_{i_m}(\theta)\rvert+\lvert\Xi^2_{f_l}(\theta)\vee\Xi^2_{f_m}(\theta)\rvert\right)}$ $JG_{10}(l,m)=\frac{1}{\lvert x\rvert}\sum_{\theta\in X}\frac{\left(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert+\left\lvert\left(1-\Xi^2_{i_l}(\theta)\right)\wedge\left(1-\Xi^2_{i_m}(\theta)\right)\right\rvert+\left\lvert\left(1-\Xi^2_{f_l}(\theta)\right)\wedge\left(1-\Xi^2_{f_m}(\theta)\right)\right\rvert\right)}{\left(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert+\left\lvert\left(1-\Xi^2_{i_l}(\theta)\right)\vee\left(1-\Xi^2_{i_m}(\theta)\right)\right\rvert+\left\lvert\left(1-\Xi^2_{f_l}(\theta)\right)\vee\left(1-\Xi^2_{f_m}(\theta)\right)\right\rvert\right)}$ $JG_{11}(l,m)=\frac{\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)\wedge\Xi^2_{t_m}(\theta)\rvert+\left\lvert\left(1-\Xi^2_{i_l}(\theta)\right)\wedge\left(1-\Xi^2_{i_m}(\theta)\right)\right\rvert+\left\lvert\left(1-\Xi^2_{f_l}(\theta)\right)\wedge\left(1-\Xi^2_{f_m}(\theta)\right)\right\rvert\right)}{\sum_{\theta\in X}\left(\lvert\Xi^2_{t_l}(\theta)\vee\Xi^2_{t_m}(\theta)\rvert+\left\lvert\left(1-\Xi^2_{i_l}(\theta)\right)\vee\left(1-\Xi^2_{i_m}(\theta)\right)\right\rvert+\left\lvert\left(1-\Xi^2_{f_l}(\theta)\right)\vee\left(1-\Xi^2_{f_m}(\theta)\right)\right\rvert\right)}$
[25]	$K_{SVNS}(l,m)=\frac{1}{n}\sum_{j=1}^{n}1-\tan\left(\frac{\pi\left(\lvert\Xi_{t_l}(\theta_j)-\Xi_{t_m}(\theta_j)\rvert+\lvert\Xi_{i_l}(\theta_j)-\Xi_{i_m}(\theta_j)\rvert+\lvert\Xi_{f_l}(\theta_j)-\Xi_{f_m}(\theta_j)\rvert\right)}{12}\right)$
[26]	$S_{1SVNS}(l^1,m^2)=\frac{1}{n}\sum_{j=1}^{n}\frac{\Xi^2_{t^1_l}(\theta)\Xi^2_{t^2_m}(\theta)+\Xi^2_{i^1_l}(\theta)\Xi^2_{i^2_m}(\theta)+\Xi^2_{f^1_l}(\theta)\Xi^2_{f^2_m}(\theta)}{\sqrt{\Xi^2_{t^1_l}(\theta)+\Xi^2_{i^1_l}(\theta)+\Xi^2_{f^1_l}(\theta)}\sqrt{\Xi^2_{t^2_m}(\theta)+\Xi^2_{i^2_m}(\theta)+\Xi^2_{f^2_m}(\theta)}},$
[27]	$NJ^1(l,m)=1-\frac{1}{3\lvert Y\rvert}\sum_V\left(\left\lvert\Xi^2_{t_l}(\rho(\theta))_V-\Xi^2_{t_m}(\rho(\theta))_V\right\rvert+\left\lvert\Xi^2_{i_l}(\rho(\theta))_V-\Xi^2_{i_m}(\rho(\theta))_V\right\rvert+\left\lvert\Xi^2_{f_l}(\rho(\theta))_V-\Xi^2_{f_m}(\rho(\theta))_V\right\rvert\right),$ $NJ^2(l,m)=1-\frac{1}{3\lvert Y\rvert}\sum_V\left(\left\lvert\Xi^2_{t_l}(\rho(\theta))_V-\Xi^2_{t_m}(\rho(\theta))_V\right\rvert-\left\lvert\Xi^2_{i_l}(\rho(\theta))_V-\Xi^2_{i_m}(\rho(\theta))_V\right\rvert-\left\lvert\Xi^2_{f_l}(\rho(\theta))_V-\Xi^2_{f_m}(\rho(\theta))_V\right\rvert\right),$ $NJ^1(l,m)=1-\frac{1}{\lvert Y\rvert}\sum_V\left(\left\lvert\Xi^2_{t_l}(\rho(\theta))_V-\Xi^2_{t_m}(\rho(\theta))_V\right\rvert\vee\left\lvert\Xi^2_{i_l}(\rho(\theta))_V-\Xi^2_{i_m}(\rho(\theta))_V\right\rvert\vee\left\lvert\Xi^2_{f_l}(\rho(\theta))_V-\Xi^2_{f_m}(\rho(\theta))_V\right\rvert\right).$

Crops are essential for many aspects of daily life. Agriculture is the main source of food for both people and animals. A significant fraction of the world's population relies on grains, such as rice, wheat, maize, and barley, as part of their main diet. The vital vitamins and nutrients necessary to maintain a balanced diet are provided by fruits, legumes, and vegetables. Crop production generates income and employment opportunities for farmers, agricultural workers, and anyone involved in agriculture [55], which benefits the economy. In addition, it supports related industries, including food packaging, processing, shipping, and marketing. Crops also contribute to the environment and development by absorbing carbon dioxide and emitting oxygen during photosynthesis, reducing greenhouse gases and mitigating climate change. Furthermore, when used in environmentally friendly agriculture, crops can contribute to soil preservation, erosion control, and water management.

Crop production involves the cultivation of plants for fiber, food, or other uses, such as grains, maize, vegetables, fruits, and feed for cattle. Soil preparation, planting, irrigation, weed control, and reaping are all parts of crop production. Several other factors influence crop growth, such as pest and disease control, water availability, soil quality, weather, and climate, crop variety, soil quality, etc. Farmers must take into account all these factors to maximize crop production. There is a research gap in achieving more accurate and precise results in the cultivation of crops with diverse parameters. Thus, this study uses NHSS formulas for distance and similarity measures [56] and the NHSS-TOPSIS technique. The similarity measure in TOPSIS improves the accuracy in ranking alternatives, and the NHS allows any characteristic (such as texture, fertility, PH values, and temperature use) to be determined with degrees of truth, indeterminacy, and falsity, providing a more realistic and comprehensive decision-making framework.

In recent years, various MCDM frameworks have incorporated uncertainty theories. For instance, in [56], a fuzzy-rough MCDM model was applied to agritourism, capturing vagueness but not neutrosophic indeterminacy. In [57], a soft-computing approach was proposed for evaluating agricultural technology using expert inputs, but it lacked hypersoft partitioning. In [58], parameter sub-attribute modelling was introduced for HSS, but without incorporating neutrosophic truthness, indeterminacy, and falsity with distance-based ranking. In [59], a neutrosophic CRITIC–COPRAS model was used for sustainable location selection without a hypersoft sub-attribute structure or sensitivity analysis. In [58], fuzzy and neutrosophic extensions were examined in decision-making, but without applying similarity-TOPSIS cases. These works focus on uncertainty modeling and soft computing in MCDM, but do not cover multi-attribute partitioning, neutrosophic indeterminacy, similarity and distance-based TOPSIS, or robustness validation. This study addresses this gap by combining a TOPSIS method of similarity-measurement with NHSS for decision-making about crop production, resulting in more robust and flexible rankings in unpredictable and complicated situations.

II. PRELIMINARIES

This section discusses some basic definitions from the literature for a better understanding of the proposed study.

A. Definition 1 (SS) [7]

Let $\Xi = \{\varsigma_1, \varsigma_2, \varsigma_3, \dots \varsigma_n\}$ be the set of alternatives and ζ be the set of parameters. Let $\varphi(\Xi)$ denote the power set of Ξ and $\chi' \subset \zeta$. Then a pair (η, χ') is called an SS [8] over Ξ as it follows the mapping: $\eta: \chi' \to \varphi(\Xi)$.

B. Definition 2 (HSS) [39]

Let a universal set $\Xi = \{\varsigma_1, \varsigma_2, \varsigma_3, \dots \varsigma_n\}$ and the power set $\varphi(\Xi)$ for Ξ. Take $R = R_1, R_2, R_3, \dots, R_n$ for $n \geq 1$, be n parameters which are well-defined, and their parametric values are the set $\chi^1, \chi^2, \chi^3, \dots, \chi^n$, respectively, with $\chi^a \cap \chi^b = \emptyset$, for $a \neq b$ and $a, b \in \{1,2,3, \dots, n\}$. An $(\eta, \chi^1, \chi^2, \chi^3, \dots, \chi^n)$ is said to be HSS [48] over Ξ where, $\eta: \chi^1, \chi^2, \chi^3, \dots, \chi^n \to \varphi(\Xi)$.

C. Definition 3 (NS) [5]

Let a universal set $\Xi = \{\varsigma_1, \varsigma_2, \varsigma_3, \dots \varsigma_n\}$ and ζ be the set of characteristics. Let $\varphi(\Xi)$ denote the power set of Ξ and $\alpha \subset \beta$. Then a mapping $T^k, I^k, F^k \to [0,1]$ is called NSS [14] and is further defined as:

$$\Gamma^k = \{< \alpha^i, (\varsigma, T^k(\varsigma), I^k(\varsigma), F^k(\varsigma)) >, \varsigma \in \Xi, \alpha^i \in \alpha\},$$

where:

$$0 \leq T^k + I^k + F^k \leq 3.$$

D. Definition 4 (NHSS) [42]

Let a universal set $\Xi = \{\varsigma_1, \varsigma_2, \varsigma_3, \dots \varsigma_n\}$ and the set of parameters ζ. Take n well-defined parameters $R = R_1, R_2, R_3, \dots, R_n$ and their corresponding attribute values $\chi^1, \chi^2, \chi^3, \dots, \chi^n$ where $\chi^a \cap \chi^b = \emptyset$, for $a \neq b$ and $a, b \in \{1,2,3, \dots, n\}$ and its relation $\chi^1 \times \chi^2 \times \chi^3 \times, \dots, \chi^n = \delta$. A pair (Γ^k, δ) is called NHSS [52] over Ξ, defined as:

$$\Gamma^k: \chi^1 \times \chi^2 \times \chi^3 \times, \dots, \chi^n \to \varphi(\Xi)$$

and:

$$\Gamma^k(\chi^1 \times \chi^2 \times \chi^3 \times, \dots, \chi^n) = \{< R^i, (\varsigma, T^k(\delta), I^k(\delta), F^k(\delta)) >, \varsigma \in \Xi, R^i \in R\}$$

where $0 \leq T^k + I^k + F^k \leq 3$.

E. Definition 5 (NHSM) [46]

Let a universal set $\Xi = \{\varsigma_1, \varsigma_2, \varsigma_3, \dots \varsigma_n\}$ and a power set $\varphi(\Xi)$ of Ξ. Take $R = R_1, R_2 \dots \dots . R_l$ for $l \geq 1$ be l well-defined parameters. Their corresponding parametric values are the sets of $\chi_1^a, \chi_2^b, \chi_3^c, \dots \dots \chi_l^z$ with relation $\chi_1^a \times \chi_2^b \times \chi_3^c \times, \dots, \chi_l^z$, where $a, b, c, \dots, z = 1,2,3, \dots, n$ and ζ be the set of parameters. The NHSS $\Gamma^k: \chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z \to \varphi(\Xi)$ can be denoted as:

$$\Gamma^k(\chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z) = \{< R^i, (\varsigma, T^k(\delta), I^k(\delta), F^k(\delta)), \varsigma \in \Xi, R^i \in R\}$$

and:

$$l \in (\chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z).$$

Let $X_{M^k} = (\chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z)$ be a relation whose characteristic functions are given as:

$$X_{M^k}: \chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z \to \varphi(\Xi).$$

This is defined as:

$$X_{M_l^k} = \{< R^i, (\varsigma, T^k(\delta), I^k(\delta), F^k(\delta)), \varsigma \in \Xi, R^i \in R\}$$

and $l \in (\chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z)$ are called NHSM.

If $B_{PQ} = X_{M_l^k}(\delta^P, \chi_Q^\theta)$, where $P = 1,2,3, \dots a$, $Q = 1,2,3, \dots, l$, $\theta = a, b, c \dots . z$, then a matrix is formed as:

$$B_{PQ} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1l} \\ b_{21} & b_{22} & b_{23} & & b_{2l} \\ \vdots & & & \ddots & \vdots \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3l} \end{pmatrix}$$

where:

$$B_{PQ} = \left(T_{\chi_Q^\theta}(\delta^P), F_{\chi_Q^\theta}(\delta^P), F_{\chi_Q^\theta}(\delta^P), \ \delta^P \in \Xi, \ \chi_Q^\theta \in \chi_1^a \times \chi_2^b \times \chi_3^c \times \dots \dots \chi_l^z\right) = \left(T_{PQ\theta}^B, I_{PQ\theta}^B, F_{PQ\theta}^B\right)$$

III. PROPOSED DISTANCE MEASURES FOR NHSS

A. Definition 6

Let $Y = Y_{\mathcal{T}}$ and $Z = Z_{\mathcal{T}}$ be the two NHSSs in which $Y_{\mathcal{T}} = (T_{\mathcal{T}}^Y, I_{\mathcal{T}}^Y, F_{\mathcal{T}}^Y)$ and $Z_{\mathcal{T}} = (T_{\mathcal{T}}^Z, I_{\mathcal{T}}^Z, F_{\mathcal{T}}^Z)$ for $\mathcal{T} = 1,2,3 \dots l$. A distance $d^t(Y,Z)$ for $t = 1,2,3$ between $Y = Y_{\mathcal{T}}$ and $Z = Z_{\mathcal{T}}$ is defined as:

$$d^1(Y,Z) = \frac{1}{3|\xi|}\sum_{\mathcal{T}}(|(T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2| + |(I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2| + |(F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2|) \quad (1)$$

$$d^2(Y,Z) = \frac{1}{3|\xi|}\sum_{\mathcal{T}}(|((T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2) - ((I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2) - ((F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2)|) \quad (2)$$

$$d^3(Y,Z) = \frac{1}{|\xi|}\sum_{\mathcal{T}}(|((T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2)\vee((I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2)\vee((F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2)|) \quad (3)$$

1) Proposition

Let $Y = Y_{\mathcal{T}}$, $Z = Z_{\mathcal{T}}$, and $X = X_{\mathcal{T}}$ be the three NHSSs in which $Y_{\mathcal{T}} = (T_{\mathcal{T}}^Y, I_{\mathcal{T}}^Y, F_{\mathcal{T}}^Y)$, $Z_{\mathcal{T}} = (T_{\mathcal{T}}^Z, I_{\mathcal{T}}^Z, F_{\mathcal{T}}^Z)$ and $X_{\mathcal{T}} = (T_{\mathcal{T}}^X, I_{\mathcal{T}}^X, F_{\mathcal{T}}^X)$ for $\mathcal{T} = 1,2,3 \ldots l$.

Then, a mapping D, written as $D: F(a) \times F(a) \to [0,1]$, is called a DM that satisfies the following conditions for Z, Y, and X and $R \subseteq F(a)$:

$D_1: 0 \leq D(Y,Z) \leq 1$,

$D_2: D(Y,Z) = 0$ If $Y = Z$,

$D_3: D(Y,Z) = D(Z,Y)$,

$D_4 = Y \subseteq Z \subseteq X$,

Then $D(Y,X) \geq D(Y,Z)$ and $D(Y,X) \geq D(Z,X)$.

B. Similarity Measures for NHSS

Let $Y = [Y_{ij}]$ and $Z = [Z_{ij}]$ be two NHSM of order $l \times m$

$$d^1(Y,Z) = \frac{1}{3|lm|}\sum_l\sum_m(|(T_{lm}^Y)^2 - (T_{lm}^Z)^2| + |(I_{lm}^Y)^2 - (I_{lm}^Z)^2| + |(F_{lm}^Y)^2 - (F_{lm}^Z)^2|) \quad (4)$$

$$d^2(Y,Z) = \frac{1}{3|lm|}\sum_l\sum_m(|((T_{lm}^Y)^2 - (T_{lm}^Z)^2) - ((I_{lm}^Y)^2 - (I_{lm}^Z)^2) - ((F_{lm}^Y)^2 - (F_{lm}^Z)^2)|) \quad (5)$$

$$d^3(Y,Z) = \frac{1}{|lm|}\sum_l\sum_m(|((T_{lm}^Y)^2 - (T_{lm}^Z)^2)\vee((I_{lm}^Y)^2 - (I_{lm}^Z)^2)\vee((F_{lm}^Y)^2 - (F_{lm}^Z)^2)|) \quad (6)$$

C. New Similarity Measures for NHSS

1) Definition 7

Let $Y = Y_{\mathcal{T}}$ and $Z = Z_{\mathcal{T}}$ be the two NHSSs in which $Y_{\mathcal{T}} = (T_{\mathcal{T}}^Y, I_{\mathcal{T}}^Y, F_{\mathcal{T}}^Y)$ and $Z_{\mathcal{T}} = (T_{\mathcal{T}}^Z, I_{\mathcal{T}}^Z, F_{\mathcal{T}}^Z)$ for $\mathcal{T} = 1,2,3 \ldots l$. A distance $S^t(Y,Z)$ for $t = 1,2,3$ between $Y = Y_{\mathcal{T}}$ and $Z = Z_{\mathcal{T}}$ is defined as:

$$S^1(Y,Z) = 1 - \frac{1}{3|\xi|}\sum_{\mathcal{T}}(|(T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2| + |(I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2| + |(F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2|) \quad (7)$$

$$S^2(Y,Z) = 1 - \frac{1}{3|\xi|}\sum_{\mathcal{T}}(|((T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2) - ((I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2) - ((F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2)|) \quad (8)$$

$$S^3(Y,Z) = 1 - \frac{1}{|\xi|}\sum_{\mathcal{T}}(|((T_{\mathcal{T}}^Y)^2 - (T_{\mathcal{T}}^Z)^2)\vee((I_{\mathcal{T}}^Y)^2 - (I_{\mathcal{T}}^Z)^2)\vee((F_{\mathcal{T}}^Y)^2 - (F_{\mathcal{T}}^Z)^2)|) \quad (9)$$

2) Proposition

Let $Y = Y_{\mathcal{T}}$, $Z = Z_{\mathcal{T}}$, and $X = X_{\mathcal{T}}$ be the three NHSSs in which $Y_{\mathcal{T}} = (T_{\mathcal{T}}^Y, I_{\mathcal{T}}^Y, F_{\mathcal{T}}^Y)$, $Z_{\mathcal{T}} = (T_{\mathcal{T}}^Z, I_{\mathcal{T}}^Z, F_{\mathcal{T}}^Z)$ and $X_{\mathcal{T}} = (T_{\mathcal{T}}^X, I_{\mathcal{T}}^X, F_{\mathcal{T}}^X)$ for $\mathcal{T} = 1,2,3,\ldots,l$. Then, a mapping $S: F(a) \times F(a) \to [0,1]$ is called an SM that satisfies the following conditions for Z, Y, and X and $R \subseteq F(a)$:

$S_1: 0 \leq S(Y,Z) \leq 1$,

$S_2: S(Y,Z) = 0$ if $Y = Z$

$S_3: S(Y,Z) = S(Z,Y)$

$S_4: Y \subseteq Z \subseteq X$

Then $S(Y,X) \geq S(Y,Z)$ and $S(Y,X) \geq S(Z,X)$.

3) Proposed Similarity Measures for NHSS

Let $Y = [Y_{ij}]$ and $Z = [Z_{ij}]$ be two NHSMs of order lm:

$$S^1(Y,Z) = 1 - \frac{1}{3|lm|}\sum_l\sum_m(|(T_{lm}^Y)^2 - (T_{lm}^Z)^2| + |(I_{lm}^Y)^2 - (I_{lm}^Z)^2| + |(F_{lm}^Y)^2 - (F_{lm}^Z)^2|) \quad (10)$$

$$S^2(Y,Z) = 1 - \frac{1}{3|lm|}\sum_l\sum_m(|((T_{lm}^Y)^2 - (T_{lm}^Z)^2) - ((I_{lm}^Y)^2 - (I_{lm}^Z)^2) - ((F_{lm}^Y)^2 - (F_{lm}^Z)^2)|) \quad (11)$$

$$S^3(Y,Z) = 1 - \frac{1}{|lm|}\sum_l\sum_m(|((T_{lm}^Y)^2 - (T_{lm}^Z)^2)\vee((I_{lm}^Y)^2 - (I_{lm}^Z)^2)\vee((F_{lm}^Y)^2 - (F_{lm}^Z)^2)|) \quad (12)$$

IV. PROPOSED ALGORITHM OF TOPSIS TECHNIQUE FOR NHSS

TOPSIS is one of the most suitable methods to address MADM problems. This section deals with the construction of the TOPSIS technique for NHSS by using distance and similarity measure formulas. The algorithm for the TOPSIS technique depends on several steps, as shown in Figure 1.

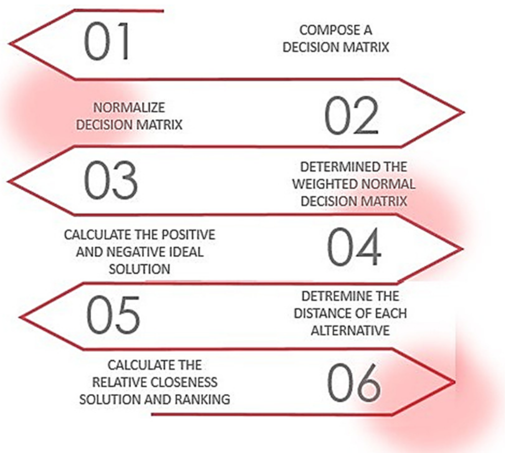

Fig. 1. Flow chart for the TOPSIS.

Consider an MADM problem based on NHSS in which $R = R^1, R^2, R^3, \ldots, R^m$ are the set of respective alternatives and the set of attributes is $m_1, m_2, m_3, \ldots, m_n$, whose corresponding attribute values are $m_1^a, m_2^b, \ldots, m_b^z$, such that $a, b, c, \ldots, z =$

$1,2,3,\dots n$. Let us consider the weight w^i of attributes m_1^z, $i = 1,2,3,\dots,n$, and $0 \le w^i \le 1$, where $\sum_{i=1}^{n} w^i = 1$. Let $(M_1, M_2, \dots, M_l)$ be the set of decision makers, and their weight being Δ^i, with $0 \le \Delta^y \le 1$ and $\sum_{i=1}^{y} \Delta^y = 1$.

Suppose the decision matrix is denoted as:

$$D_{ij}^{y} = \left(T_{ijk}^{y}, I_{ijk}^{y}, F_{ijk}^{y}\right)$$

where $i = 1,2,3\dots,m$, $j = 1,2,3,\dots.,n$, $k = a,b,c,\dots,z$, and $T_{ijk}^{y}, I_{ijk}^{y}, F_{ijk}^{y} \in [0,1]$, satisfying the condition $0 \le T_{ijk}^{y}, I_{ijk}^{y}, F_{ijk}^{y} \le 3$.

The alternative selection can be obtained by using the following steps.

1) Step 1 - Creating the Decision Matrix and Determining the Weights

Suppose the decision matrix $[D_{ij}^{y}]_{a\times b}$. For $y = 1,2,3,\dots s$, all the decision matrices D_{ij}^{y} are averaged, finding the ideal matrix $[D''_{ij\,a\times b}]$, where

$$D''_{ij} = T''_{m_1^k}, I''_{m_1^k}, F''_{m_1^k} =$$

$$\left(1 - \sum_{y=1}^{s}\left(1 - T_{m_1^k}^{y}\right)^{\frac{1}{s}}, \sum_{y=1}^{s}\left(I_{m_1^k}^{y}\right)^{\frac{1}{s}}, \sum_{y=1}^{s}\left(F_{m_1^k}^{y}\right)^{\frac{1}{s}}\right)$$

Now, the SM corresponding to each decision and ideal matrix is found to determine the weights using

$$S^1\left(D_{ij}^{y}, D''_{ij}\right) = 1 - \frac{1}{3|ab|}\sum_{i}^{a}\sum_{j}^{b}\left(\left|(T_{m_1^k}^{y})^2 - \left(T''_{m_1^k}\right)^2\right| + \left|(I_{m_1^k}^{y})^2 - \left(I''_{m_1^k}\right)^2\right| - \left|(F_{m_1^k}^{y})^2 - \left(F''_{m_1^k}\right)^2\right|\right)$$

Using this equation, the weights $\Delta^y(y = 1,2,\dots,s)$ concerting t decision-makers can be calculated as:

$$\Delta^y(y = 1,2,\dots,s) = S\left(D_{ij}^{y}, D''_{ij}\right)$$

where, $0 \le \Delta^y \le 1$ and $\sum_{y=1}^{s} = 1$.

2) Step 2 - Normalize the NHSS Decision Matrix

Now, create an NHSS decision matrix with each row representing a decision and every column representing a criterion. To verify that the criteria are on the same scale, normalize the values in the matrix. Each element in the matrix $\left[D_{ij}\right]_{a\times b}$ is calculated as:

$$\left[D_{ij}\right]_{a\times b} = (1 - \prod_{y=1}^{s}\left(1 - T_{m_1^k}^{y}\right)^{\Delta^y},$$

$$\prod_{y=1}^{s}\left(I_{m_1^k}^{y}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(F_{m_1^k}^{y}\right)^{\Delta^y}$$

3) Step 3 - Aggregate the Weight of Attributes

Decision-makers may believe that not all attributes are equally significant during the decision-making process, viewing the attribute weights differently. To obtain an anonymous evaluation, the opinions of all the decision-makers regarding the weight of each of the selected attributes must be aggregated. The weight w^i of the attributes is calculated as:

$$w^i = (T_m, I_m, F_m)(1 - \prod_{y=1}^{s}\left(1 - T_{m_1^k}^{y}\right)^{\Delta^y},$$

$$\prod_{y=1}^{s}\left(I_{m_1^k}^{y}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(F_{m_1^k}^{y}\right)^{\Delta^y}$$

4) Step 4 - Determine the Weighted Aggregated NHSS Decision Matrix

The calculated weights in the last step corresponding to each of the individual attributes can be implemented in each row of the normalized NHSS decision matrix. Then, a weighted aggregated NHSS decision matrix can be found, such as:

$$[D_{ij}^{y}]_{a\times b}=\begin{bmatrix} T_{m_1^a}^{y}(r^1), I_{m_1^a}^{y}(r^1), F_{m_1^a}^{y}(r^1) & T_{m_2^a}^{y}(r^1), I_{m_2^a}^{y}(r^1), F_{m_2^a}^{y}(r^1) & \dots & T_{m_b^a}^{y}(r^1), I_{m_b^a}^{y}(r^1), F_{m_b^a}^{y}(r^1) \\ T_{m_1^a}^{y}(r^2), I_{m_1^a}^{y}(r^2), F_{m_1^a}^{y}(r^2) & T_{m_2^a}^{y}(r^2), I_{m_2^a}^{y}(r^2), F_{m_2^a}^{y}(r^2) & \dots & T_{m_b^a}^{y}(r^2), I_{m_b^a}^{y}(r^2), F_{m_b^a}^{y}(r^2) \\ \vdots & \vdots & \ddots & \vdots \\ T_{m_1^a}^{y}(r^a), I_{m_1^a}^{y}(r^a), F_{m_1^a}^{y}(r^a) & T_{m_2^a}^{y}(r^a), I_{m_2^a}^{y}(r^a), F_{m_2^a}^{y}(r^a) & \dots & T_{m_b^a}^{y}(r^a), I_{m_b^a}^{y}(r^a), F_{m_b^a}^{y}(r^a) \end{bmatrix},$$

$$[D''_{ij}]_{a\times b}=\begin{bmatrix} T''_{m_1^a}(r^1), I''_{m_1^a}(r^1), F''_{m_1^a}(r^1) & T''_{m_2^a}(r^1), I''_{m_2^a}(r^1), F''_{m_2^a}(r^1) & \dots & T''_{m_b^a}(r^1), I''_{m_b^a}(r^1), F''_{m_b^a}(r^1) \\ T''_{m_1^a}(r^2), I''_{m_1^a}(r^2), F''_{m_1^a}(r^2) & T''_{m_2^a}(r^2), I''_{m_2^a}(r^2), F''_{m_2^a}(r^2) & \dots & T''_{m_b^a}(r^2), I''_{m_b^a}(r^2), F''_{m_b^a}(r^2) \\ \vdots & \vdots & \ddots & \vdots \\ T''_{m_1^a}(r^a), I''_{m_1^a}(r^a), F''_{m_1^a}(r^a) & T''_{m_2^a}(r^a), I''_{m_2^a}(r^a), F''_{m_2^a}(r^a) & \dots & T''_{m_b^a}(r^a), I''_{m_b^a}(r^a), F''_{m_b^a}(r^a) \end{bmatrix},$$

$$[D_{ij}]_{a\times b}=\begin{bmatrix} T_{m_1^a}(r^1), I_{m_1^a}(r^1), F_{m_1^a}(r^1) & T_{m_2^a}(r^1), I_{m_2^a}(r^1), F_{m_2^a}(r^1) & \dots & T_{m_b^a}(r^1), I_{m_b^a}(r^1), F_{m_b^a}(r^1) \\ T_{m_1^a}(r^2), I_{m_1^a}(r^2), F_{m_1^a}(r^2) & T_{m_2^a}(r^2), I_{m_2^a}(r^2), F_{m_2^a}(r^2) & \dots & T_{m_b^a}(r^2), I_{m_b^a}(r^2), F_{m_b^a}(r^2) \\ \vdots & \vdots & \ddots & \vdots \\ T_{m_1^a}(r^a), I_{m_1^a}(r^a), F_{m_1^a}(r^a) & T_{m_2^a}(r^a), I_{m_2^a}(r^a), F_{m_2^a}(r^a) & \dots & T_{m_b^a}(r^a), I_{m_b^a}(r^a), F_{m_b^a}(r^a) \end{bmatrix},$$

$$[D_{ij}^{w}] = \left(T_{m_1^k}^{w}, I_{m_1^k}^{w}, F_{m_1^k}^{w}\right) = ((T_{m_i^k}, T_{m_i}), (I_{m_i^k} + I_{m_i} - I_{m_i^k}, I_{m_i}), (F_{m_i^k} + F_{m_i} - F_{m_i^k}, F_{m_i})$$

5) Step 5 - Calculate the PIS $D_{ij}^{w^+}$ and NIS $D_{ij}^{w^-}$

The positive $D_{ij}^{w^+}$ and negative $D_{ij}^{w^-}$ ideal solutions can be identified using the score and accuracy functions. The greatest and worst consequences of the possible result of these ideal solutions will be utilized to assess the alternatives being explored. The PIS for NHSS can be determined as:

$$D_{ij}^{w^+} = \left(T_{m_1^k}^{w^+}, I_{m_1^k}^{w^+}, F_{m_1^k}^{w^+}\right) = (\max(T_{m_1^k}^{w}), \min(I_{m_1^k}^{w}), \min(F_{m_1^k}^{w}))$$

and:

$$D_{ij}^{w^+} = \left(T_{m_1^k}^{w^+}, I_{m_1^k}^{w^+}, F_{m_1^k}^{w^+}\right) = (\min(T_{m_1^k}^{w}), \max(I_{m_1^k}^{w}), \max(F_{m_1^k}^{w})),$$

Likewise, the NIS for NHSS is given as:

$$D_{ij}^{w^-} = \left(T_{m_1^k}^{w^-}, I_{m_1^k}^{w^-}, F_{m_1^k}^{w^-}\right) = (\min(T_{m_1^k}^{w}), \max(I_{m_1^k}^{w}), \max(F_{m_1^k}^{w})),$$

and:

$$D_{ij}^{w^-} = \left(T_{m_1^k}^{w^-}, I_{m_1^k}^{w^-}, F_{m_1^k}^{w^-}\right) = (\max(T_{m_1^k}^{w}), \min(I_{m_1^k}^{w}), \min(F_{m_1^k}^{w}))$$

6) Step 6 - Determine the Distances

Calculate the distance of each alternative for PIS and NIS, such as:

$$D^{i^+}(D_{ij}^{w}, D_{ij}^{w^+}) = \frac{1}{3|ab|}\sum_i^n\sum_j^b\left(\left|(T_{m_1^k}^{w})^2 - \left(T_{m_1^k}^{w^+}\right)^2\right| + \left|(I_{m_1^k}^{w})^2 - \left(I_{m_1^k}^{w^+}\right)^2\right| - \left|(F_{m_1^k}^{w})^2 - \left(F_{m_1^k}^{w^+}\right)^2\right|\right),$$

Similarly, we obtain:

$$D^{i^-}(D_{ij}^{w}, D_{ij}^{w^-}) = \frac{1}{3|ab|}\sum_i^n\sum_j^b\left(\left|(T_{m_1^k}^{w})^2 - \left(T_{m_1^k}^{w^-}\right)^2\right| + \left|(I_{m_1^k}^{w})^2 - \left(I_{m_1^k}^{w^-}\right)^2\right| - \left|(F_{m_1^k}^{w})^2 - \left(F_{m_1^k}^{w^-}\right)^2\right|\right).$$

7) Step 7 - Calculate Relative Closeness to the Ideal Solution

By using the distance measured in the previous step, the relative closeness corresponding to the ideal solution for ranking the alternatives can be calculated for $i = 1, 2, \dots, n$.

$$RC^i = \frac{D^{i^-}(D_{ij}^{w}, D_{ij}^{w^-})}{max^i\{D^{i^-}(D_{ij}^{w}, D_{ij}^{w^-})\}} - \frac{D^{i^+}(D_{ij}^{w}, D_{ij}^{w^+})}{max^i\{D^{i^+}(D_{ij}^{w}, D_{ij}^{w^+})\}}$$

The alternative with the greatest relative closeness is deemed the best option.

V. APPLICATION OF NHSS-TOPSIS TECHNIQUE

The term soil refers to an independent natural body with a distinctive morphology from the soil surface to the parent material as reflected by the soil profile. The elements of soil formation include climate, organisms, parent material, relief, and time. People are becoming more aware that soil resources are very important for growing good crops. To monitor alterations in soil quality, criteria are still required to understand how soil quality is changing. Soil is a biologically active, permeable substance that has grown in the earth's crust's top layer. Soil is one of the fundamental substrates of life on earth, contributing as a source of water and nutrients, a filter for toxic waste, an environment for their decomposition, and an integral component of the carbon and other element cycles that occur throughout the planet's ecosystem. Soil changes due to weathering processes produced by topographical, geologic, biological, and climatic causes. The main goal in this work was to compare variations in soil characteristics and crop productivity. As previously mentioned, among all the problems, the most challenging is to define the relation for the function:

$$\eta: \zeta_a^1 \times \zeta_b^2 \times \zeta_c^3 \times \zeta_d^4 \to \varphi(P)$$

as $\eta: \zeta_a^1 \times \zeta_b^2 \times \zeta_c^3 \times \zeta_d^4 = \eta: \zeta_1^1 \times \zeta_2^2 \times \zeta_2^3 \times \zeta_1^4 = (Sandy(S), 6.5 - 7.5(PH), Acquired\ (A), 18 - 24\ (T))$ is the actual requirement for the best production of maize. Three locations are selected $P = \{\rho^1, \rho^2, \rho^3\}$ and examined to determine which can offer the best production of maize. Two decision-makers, M_k^1, M_k^2, select the most appropriate requirements for maize production. Decision-makers provide vital expertise in NHSM separately.

Take the most important factors that affect the production of crops $\zeta = \{\zeta^1(\text{soil textured})\}$, $\zeta^2(\text{PH} - \text{values})$, $\zeta^3(\text{soil fertility})$, $\zeta^4(\text{temperature})\}$ that bifurcate even further into the most suitable criteria.

$$\zeta_a^1 = \{sandy, clay, slit, loam\},\ a = 1,2,3,4$$

$$\zeta_b^2 = \{5 - 7, 6.5 - 7.5, 6.5 - 9, over\ 7.5\},\ b = 1,2,3,4$$

$$\zeta_c^3 = \{natural, acquired\},\ c = 1,2,3,4$$

$$\zeta_d^4 = \{18 - 24, 24 - 31, 21 - 27\},\ d = 1,2,3,4$$

Now, define the relation for the function:

$$\eta: \zeta_a^1 \times \zeta_b^2 \times \zeta_c^3 \times \zeta_d^4 \to \varphi(P),$$

$$\eta: \zeta_a^1 \times \zeta_b^2 \times \zeta_c^3 \times \zeta_d^4 = \eta: \zeta_1^1 \times \zeta_2^2 \times \zeta_2^3 \times \zeta_1^4 = (Sandy(S),\ 6.5 - 7.5(PH), Acquired\ (A), 18 - 24\ (T))$$

which is the actual requirement for the best production of maize. Attributes chosen by each decision-maker are prioritized as M_k^1, M_k^2 in (1) and (2):

1) Step 1 - Creating the Decision Matrix and Determining the Weights

For $y = 1,2$, average all the decision matrices M_k^1, M_k^2 and construct an ideal matrix as:

$$D_{ij}'' = (T_{m_1^k}'', I_{m_1^k}'', F_{m_1^k}'') = \left(1 - \prod_{y=1}^s\left(1 - T_{m_1^k}^{y}\right)^{\frac{1}{s}}, \prod_{y=1}^s\left(I_{m_1^k}^{y}\right)^{\frac{1}{s}}, \prod_{y=1}^s\left(F_{m_1^k}^{y}\right)^{\frac{1}{s}}\right)$$

For $i = 1, 2, 3$ and $k = a, b, c, d$ such that $a = 1$, $b = 2$, $c = 2$, and $d = 1$. For $i = 1$, $k = a = 1$

$w''_{11} = \left(T''_{m_1^1}, I''_{m_1^1}, F''_{m_1^1}\right) = \left(1 - \prod_{y=1}^{2}\left(1 - T^{y}_{m_1^1}\right)^{\frac{1}{2}}, \prod_{y=1}^{2}\left(I^{y}_{m_1^1}\right)^{\frac{1}{2}}, \prod_{y=1}^{2}\left(F^{y}_{m_1^1}\right)^{\frac{1}{2}}\right)$

which gives:

w''_{11}=(.76,.24,.32).

2) Step 2 - Normalize the NHSS Decision Matrix

Construct an NHSS decision matrix with each row representing a decision and each column representing a criterion. So, the NHSS decision matrix D_{ij} is given as:

$$\boldsymbol{M_k^1} = \begin{bmatrix} (\boldsymbol{S},(\mathbf{0.7},\mathbf{0.2},\mathbf{0.2})) & (\boldsymbol{PH},(\mathbf{0.5},\mathbf{0.4},\mathbf{0.5})) & (\boldsymbol{A},(\mathbf{0.8},\mathbf{0.3},\mathbf{0.1})) & (\boldsymbol{T},(\mathbf{0.3},\mathbf{0.3},\mathbf{0.4})) \\ (\boldsymbol{S},(\mathbf{0.7},\mathbf{0.1},\mathbf{0.5})) & (\boldsymbol{PH},(\mathbf{0.8},\mathbf{0.1},\mathbf{0.2})) & (\boldsymbol{A},(\mathbf{0.2},\mathbf{0.3},\mathbf{0.2})) & (\boldsymbol{T},(\mathbf{0.6},\mathbf{0.4},\mathbf{0.7})) \\ (\boldsymbol{S},(\mathbf{0.6},\mathbf{0.3},\mathbf{0.3})) & (\boldsymbol{PH},(\mathbf{0.7},\mathbf{0.4},\mathbf{0.5})) & (\boldsymbol{A},(\mathbf{0.7},\mathbf{0.1},\mathbf{0.3})) & (\boldsymbol{T},(\mathbf{0.6},\mathbf{0.3},\mathbf{0.4})) \end{bmatrix}$$

$$M_k^2 = \begin{bmatrix} (S,(0.8,0.3,0.5)) & (PH,(0.6,0.3,0.6)) & (A,(0.7,0.1,0.2)) & (T,(0.5,0.5,0.6)) \\ (S,(0.9,0.2,0.2)) & (PH,(0.7,0.2,0.1)) & (A,(0.7,0.2,0.4)) & (T,(0.7,0.4,0.5)) \\ (S,(0.6,0.3,0.3)) & (PH,(0.7,0.5,0.5)) & (A,(0.2,0.3,0.5)) & (T,(0.2,0.5,0.7)) \end{bmatrix}$$

Each element D_{ij} in the matrix $[D_{ij}]_{a\times b}$ is calculated as:

$[D_{ij}]_{a\times b} = (1 - \prod_{y=1}^{s}\left(1 - T^{y}_{m_1^k}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(I^{y}_{m_1^k}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(F^{y}_{m_1^k}\right)^{\Delta^y})$

The normalized NHSS decision matrix is given as:

$M_k^1 \rightarrow \left(S,(0.7,0.2,0.3)\right)\left(PH,(0.7,0.3,0.4)\right)$

$(A,(0.6,0.2,0.2))\ (T,(0.5,0.3,0.5))$

$M_k^2 \rightarrow \left(S,(0.8,0.3,0.3)\right)\left(PH,(0.7,0.3,0.4)\right)$

$(A,(0.5,0.2,0.4))\ (T,(0.5,0.5,0.6)$

3) Step 3 - Create a Normalized Weighted Attribute

The weights w^i of attributes for $i = 1,2$ are calculated as:

$w^i = (T_m, I_m, F_m)(1 - \prod_{y=1}^{s}\left(1 - T^{y}_{m_1^k}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(I^{y}_{m_1^k}\right)^{\Delta^y}, \prod_{y=1}^{s}\left(F^{y}_{m_1^k}\right)^{\Delta^y}).$

Using prioritized decision matrices selected by the decision-maker gives:

$w^1 = (0.7547,0.2446,0.3),$

$w^2 = (0.7,0.3,0.4),$

$w^3 = (0.5531,0.2,0.2822),$

$w^4 = (0.5,0.3867,0.5474).$

4) Step 4 - Determine the Weighted Aggregated NHSS Decision Matrix

Each row of the normalized NHSS decision matrix can incorporate the calculated weights from the previous step for each of the individual attributes.

$[D_{ij}^{w}] = \left(T^{w}_{m_1^k}, I^{w}_{m_1^k}, F^{w}_{m_1^k}\right) = ((T_{m_i^k}, T_{m_i}), (I_{m_i^k} + I_{m_i} - I_{m_i^k}, I_{m_i}), (F_{m_i^k} + F_{m_i} - F_{m_i^k}, F_{m_i})$

After substituting weighted attributes in an aggregated decision matrix, a decision matrix Ω^w is obtained:

$$[D_{ij}]_{a\times b} = \begin{bmatrix} T_{m_1^a}(r^1), I_{m_1^a}(r^1), F_{m_1^a}(r^1) & T_{m_2^a}(r^1), I_{m_2^a}(r^1), F_{m_2^a}(r^1) & \dots & T_{m_b^a}(r^1), I_{m_b^a}(r^1), F_{m_b^a}(r^1) \\ T_{m_1^a}(r^2), I_{m_1^a}(r^2), F_{m_1^a}(r^2) & T_{m_2^a}(r^2), I_{m_2^a}(r^2), F_{m_2^a}(r^2) & \dots & T_{m_b^a}(r^2), I_{m_b^a}(r^2), F_{m_b^a}(r^2) \\ \vdots & \vdots & \ddots & \vdots \\ T_{m_1^a}(r^a), I_{m_1^a}(r^a), F_{m_1^a}(r^a) & T_{m_2^a}(\mathrm{r}^a), I_{m_2^a}(r^a), F_{m_2^a}(r^a) & \dots & T_{m_b^a}(r^a), I_{m_b^a}(r^a), F_{m_b^a}(r^a) \end{bmatrix}$$

$$\Omega = \begin{bmatrix} (S,(0754,0.245,0.315)) & (PH,(0.552,0.347,0.577)) & (A,(0.75,0.174,0.14)) & (T,(0.408,0.387,0.489)) \\ (S,(0.826,0.141,0.317)) & (PH,(0.755,0.1411,0.142)) & (A,(0.509,0.24,0.282)) & (T,(0.653,0.4,0.592)) \\ (S,(0.6,0.3,0.3)) & (PH,(0.7,0.447,0.5)) & (A,(0.512,0.173,0.387)) & (T,(0.435,0.387,0.528)) \end{bmatrix}$$

$$\Omega^{w} = \begin{bmatrix} (S,(0.569,0.429,0.521)) & (PH,(0.387,0.543,0.729)) & (A,(0.481,0.339,0.383)) & (T,(0.204,0.624,0.769)) \\ (S,(0.624,0.351,0.522)) & (PH,(0.529,0.398,0.485)) & (A,(0.281,0.396,0.485)) & (T,(0.327,0.632,0.815)) \\ (S,(0.453,0.471,0.51)) & (PH,(0.49,0.613,0.7)) & (A,(0.283,0.338,0.559)) & (T,(0.218,0.624,0.786)) \end{bmatrix}$$

5) Step 5 - Calculate the Ideal Solution

The PIS for NHSS is calculated as follows:

$$\Omega^{w^+} = \begin{bmatrix} \left(S,(0.624,0.351,0.51)\right), \left(PH,(0.49,0.398,0.485)\right), \\ \left(A,(0.418,0.338,0.383)\right), \left(T,(0.327,0.624,0.769)\right) \end{bmatrix}$$

Similarly, the NIS for NHSS is

$\Omega^{w^-} =$
$[(S,(0.453,0.471,0.522)),(PH,(0.387,0.613,0.728))$
$(A,(0.281,0.396,0.559)),(T,(0.204,0.632,0.815))]$

6) Step 6 - Determine the Distances

Calculate the distance of each alternative for PIS and NIS, such as:

$$D^{i^+}(D_{ij}^w, D_{ij}^{w^+}) = \frac{1}{3|ab|}\sum_i^n \sum_j^b \left(\left| (T_{m_1^k}^w)^2 - \left(T_{m_1^k}^{w^+}\right)^2 \right| + \left| (I_{m_1^k}^w)^2 - \left(I_{m_1^k}^{w^+}\right)^2 \right| - \left| (F_{m_1^k}^w)^2 - \left(F_{m_1^k}^{w^+}\right)^2 \right| \right)$$

After substituting:

$D^{1^+}(\Omega_1^w, \Omega^+) = 0.1813$, $D^{2+}(\Omega_1^w, \Omega^+) = 0.0905$, $D^{3^+}(\Omega_1^w, \Omega^+) = 0.2753$

Similarly:

$$D^{i^-}(D_{ij}^w, D_{ij}^{w^-}) = \frac{1}{3|ab|}\sum_i^n \sum_j^b \left(\left| (T_{m_1^k}^w)^2 - \left(T_{m_1^k}^{w^-}\right)^2 \right| + \left| (I_{m_1^k}^w)^2 - \left(I_{m_1^k}^{w^-}\right)^2 \right| - \left| (F_{m_1^k}^w)^2 - \left(F_{m_1^k}^{w^-}\right)^2 \right| \right).$$

where:

$D^{1^-}(\Omega_1^w, \Omega^-) = 0.1562,$

$D^{2-}(\Omega_1^w, \Omega^-) = 0.2669$

$D^{3^-}(\Omega_1^w, \Omega^-) = 0.0622$

7) Step 7 - Calculate the Relative Closeness to the Ideal Solution

By using the distance measured in the previous step, the relative closeness corresponding to the ideal solution is calculated for ranking the alternatives.

$$\mathcal{R}_{\rho^i} = \frac{D^{i^-}(D_{ij}^w, D_{ij}^{w^-})}{max^i\{D^{i^-}(D_{ij}^w, D_{ij}^{w^-})\}} - \frac{D^{i^+}(D_{ij}^w, D_{ij}^{w^+})}{max^i\{D^{i^+}(D_{ij}^w, D_{ij}^{w^+})\}}$$

which gives:

$$\mathcal{R}_{\rho^1} = \frac{0.1562}{0.2669} - \frac{0.1813}{0.0905} = -1.1481$$

$$\mathcal{R}_{\rho^2} = \frac{0.2669}{0.2669} - \frac{0.0905}{0.0905} = 0$$

$$\mathcal{R}_{\rho^3} = \frac{0.622}{0.2669} - \frac{0.2753}{0.0905} = -2.8089$$

VI. RESULTS, DISCUSSION, AND COMPARATIVE ANALYSIS WITH EXISTING APPROACHES

Three different locations $P = \{\rho^1, \rho^2, \rho^3\}$ are examined to select the best for the production of maize. The alternative with the greatest relative closeness is $\rho^2 > \rho^1 > \rho^3$, which demonstrates that location ρ^2 is the best for the production of crops (maize), as shown in Figure 2. The proposed TOPSIS algorithm for NHSS is used to rank alternatives in the NHSS environment, and the results are compared to several current decision-making approaches, as shown in Table II. The final ranking order is similar to the suggested technique. However, it should be noted that the computational technique of the suggested approach is fundamentally different from that of the existing approaches. The accuracy and durability of crop selection decision-making under uncertain circumstances were improved by the suggested NHSS TOPSIS model, showing better handling of complicated agricultural data and increased ranking stability. When compared to conventional MCDM techniques, the proposed method produced more reliable and efficient results in general.

For instance, in [54], the TOPSIS approach was used based on the Hamming distance to rank the alternatives. This study used the distance and SM formulas [46] for the NHSS TOPSIS technique to rank the alternatives. The results further validate that the proposed NHSS-TOPSIS framework offers clearer and more consistent crop rankings by efficiently handling multi-parameter uncertainty. This outcome supports the practical usefulness and innovative strength of the model within real agricultural decision-making scenarios.

TABLE II. COMPARATIVE ANALYSIS OF NHSS-TOPSIS WITH EXISTING APPROACHES

Method	Final Ranking	Best alternatives
[51]	$\rho^2 > \rho^1 > \rho^3$	ρ^2
[54]	$\rho^2 > \rho^1 > \rho^3$	ρ^2
[60]	$\rho^2 > \rho^3 > \rho^1$	ρ^2
[61]	$\rho^2 > \rho^1 > \rho^3$	ρ^2
[62]	$\rho^2 > \rho^1 > \rho^3$	ρ^2
[63]	$\rho^2 > \rho^1 > \rho^3$	ρ^2
Proposed technique	$\rho^2 > \rho^1 > \rho^3$	ρ^2

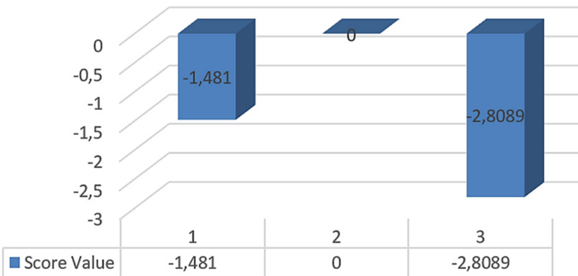

Fig. 2. Ranking of the alternatives.

VII. CONCLUSION

This study discussed some distance and SM formulas for NHSS with the help of aggregate operators and used them to extend the TOPSIS technique for MCDM. The proximity coefficient was created to evaluate the alternatives using a well-established technique. The proposed NHSS-TOPSIS framework provides an intriguing and versatile method for addressing MCDM difficulties in a variety of disciplines, such as manufacturing frameworks, supplier selection, and various management systems. The prospects of further exploration into various NHSS environments emphasize their applicability to a wide range of decision-making scenarios. Furthermore, to verify the suggested technique, a practical example for the selection of the best location for the production of crops (maize) was discussed. Decision-making in crop production involves an adaptive and holistic approach that takes into account both short-term goals and long-term sustainability.

In the future, the approaches considered could be employed in case studies with various attributes that are further subdivided into many order preference problems. This work is also extensible to many of the defined distance and similarity measure formulas for HSS hybrids, such as IFHSs, bi-polar HSs, m-polar HSs, Pythagorean HSs (with their fuzzy, intuitionistic, and neutrosophic hybrids), and many more.

ACKNOWLEDGMENT

The authors extend their appreciation to the support provided by the University of Business and Technology, Jeddah, Saudi Arabia.

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