Topology-Aware Robustness Prediction: K-Hop Entropy Analysis of D-Wave Zephyr Architectures

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ABSTRACT

The physical connectivity of quantum processors significantly affects the performance and robustness of embedded quantum algorithms, particularly in architectures like D-Wave's Zephyr. This work examines the use of K-hop Entropy Metric (KHEM) as a topology-aware metric to predict the stability of logical-to-physical qubit embeddings, eliminating the need for explicit noise models. KHEM quantifies the degree of heterogeneity within the k-hop neighborhood of each node. This provides insights into the local structural variability that influences the length of the embedding chain and the physical separation of the logical qubits. The present study analyzed simulated embeddings of Erdos–Renyi (ER) and complete logical graphs onto a Zephyr-8 architecture to assess the relationship between KHEM, average chain length, and hop-distance distributions. Jensen–Shannon Divergence (JSD) was then used to compare k = 2 and k = 3 distributions, while KHEM was shown to provide a fast, topology-driven pre-filter for robustness estimation on Zephyr-8.

Keywords-quantum annealing; Zephyr topology; k-hop entropy metric; robustness prediction; Jensen–Shannon divergence

I. INTRODUCTION

The physical connectivity of quantum processors significantly impacts the efficiency with which logical problems can be embedded and executed on real hardware. This is especially true for quantum annealers, like D-Wave systems, where a logical problem graph must be minimally embedded into a sparse hardware topology. In these settings, creating chains—sets of physical qubits that collectively represent a single logical variable—is unavoidable. Both chain length and inter-chain separation directly impact solution quality and robustness under noise. Consequently, topology-aware metrics that quickly estimate embedding difficulty and robustness without resorting to full noise simulations are valuable for algorithm designers and hardware architects. The minor-embedding theory and heuristics for annealing hardware have been examined. It has been shown that arbitrary Ising or QUBO problems can be represented on sparse physical graphs via minor embeddings. The roles of chain construction, parameter setting, and fault tolerance have also been clarified [1-3]. Methods to improve embeddings by enlarging subproblems have been demonstrated on D-Wave hardware, reinforcing the practical importance of embedding strategies for real-world optimization [4]. Subsequent developments have introduced scalable hardware topologies, transitioning from Chimera to Pegasus and, more recently, Zephyr. These architectures increase the degree and reduce the effective distance between variables. Consequently, embedding overheads are alleviated, and performance improves on larger, denser logical graphs [5, 6]. Large-scale demonstrations have further highlighted how hardware connectivity shapes emergent quantum behavior, including topological effects in lattices of thousands of qubits [7]. Practical embedding tools (e.g., Minorminer) and modeling libraries (e.g., Dwave-NetworkX) have become standard for generating feasible embeddings and analyzing their quality on a large scale [8]. Several algorithmic approaches have been proposed to efficiently exploit Chimera and Pegasus connectivity [9], and the robustness of embeddings in faulty or broken Chimera graphs [6]. Authors in [10] observed that longer chains and larger physical separations between logically adjacent variables correlate with poorer solution quality and higher error susceptibility on current devices. Beyond classical metrics, such as average chain length or mean hop distance, there is a growing interest in structural descriptors that capture the local heterogeneity of the hardware graph itself and its interaction with a problem's logical structure. Entropy-based neighborhood measures are well-suited for this purpose. The KHEM is defined as the entropy of the normalized degree distribution within a k-hop neighborhood, quantifying the local structural heterogeneity observed from a node. Authors in [11-13] analyzed D-Wave hardware families using k-hop metrics and reported systematic differences across Chimera, Pegasus, and Zephyr, which typically exhibited higher local heterogeneity at comparable scales. These results suggest that KHEM could serve as a topology-aware signal for embedding robustness. If a logical node is mapped into a region with highly heterogeneous k-hop neighborhoods, then the corresponding chains and inter-chain couplings may become more dispersed in physical space, increasing vulnerability to noise and parameter misspecification. This work makes three contributions: a KHEM-based robustness proxy is formalized for embeddings on Zephyr architectures by relating node- and edge-level k-hop entropies to standard embedding quality indicators, such as average chain length and chain-chain hop statistics; the analysis is extended with a JSD [12, 14] comparison between $k = 2$ and $k = 3$ neighborhood distributions to quantify sensitivity to neighborhood radius, offering a compact measure of how "stable" the local structure remains as the horizon grows and empirical validation through large-scale, simulated embeddings of ER [15] graphs and complete graphs onto Zephyr. These embeddings show that KHEM correlates with hop-based measures, providing a fast, topology-driven pre-filter before costly, noise-aware studies. The findings suggest that KHEM can predict which embeddings are more likely to be robust on Zephyr and, more broadly, assist in selecting or designing hardware topologies that better fit target logical graph families [16, 17]. While this study applies the KHEM to evaluate topology-driven robustness in D-Wave Zephyr architectures, similar structural metrics could be examined in other quantum and post-quantum domains. Potential application areas include quantum encryption algorithms [18], the optimization of quantum key distribution protocols [19], and the design of secure post-quantum messaging frameworks [20], where robustness against noise, topology constraints, and emerging computational threats are critical. In these contexts, system stability and resilience are conceptually similar to the topology-driven robustness analyzed in this work. These cross-domain similarities further underscore the potential relevance of structural, entropy-based measures. This study focuses on advancing KHEM from a descriptive to a predictive tool for D-Wave Zephyr embeddings. While earlier works by the author introduced KHEM as a descriptive tool for analyzing the structural heterogeneity of D-Wave processor families, this study takes the approach further by developing a predictive framework. Specifically, KHEM is systematically linked to standard robustness proxies, such as average chain length and hop-distance distributions, extended with JSD analysis to evaluate stability across neighborhood radii and validated through regression models with out-of-sample performance.

II. MODEL METHODOLOGY

The proposed framework for topology-aware robustness prediction in D-Wave Zephyr quantum architectures based on KHEM and its extensions, is described, and the methodology combines physical topology modeling, generation of a logical problem graph, minor embedding, and extraction of robustness-related metrics.

A. Overview

This work's central hypothesis is that the structural relationship between a physical qubit topology and an embedded logical problem graph can be characterized quantitatively using k-hop entropy and related metrics. Specifically, KHEM can indicate how well a given physical layout preserves the locality of logical interactions and predict embedding robustness without the need for explicit noise simulations. The methodology comprises five steps: Generating the physical topology by modeling the Zephyr architecture with a configurable size parameter, m; generating the logical graph by creating target problem graphs with a controllable size, n, and density, p, (ER or complete graph); minor-embedding search, using the Minorminer heuristic algorithm to map logical qubits to chains of physical qubits; KHEM and related metric computation, by measuring the structural heterogeneity of neighborhoods in the physical topology relative to the embedding; and robustness-oriented analysis, by applying JSD between k-hop distributions, and correlation analysis with chain lengths.

B. Physical Topology: Zephyr Architecture

The Zephyr topology is the latest in D-Wave's family of Chimera–Pegasus–Zephyr superconducting Quantum Processing Unit (QPU) architectures [8, 9]. It is parameterized by an integer m that determines the number of unit cells per side. The connectivity graph is generated via the dwave_networkx.zephyr_graph(m) function, producing a fixed-degree lattice with improved qubit output and connectivity compared to the Pegasus architecture. For this study, experiments were conducted exclusively on a Zephyr-8 instance to balance computational cost with the accuracy of Zephyr structural characteristics.

C. Logical Graphs

Logical graphs show the abstract problem that needs to be solved by the quantum annealer. There are two main types: Complete Graph (K_n): a maximally dense graph, modeling fully connected QUBOs, and the Erdős–Rényi graph $G(n,p)$: a random graph in which each edge appears independently with probability p, allowing for density control. The choice of n and p is crucial, because a larger n typically increases the average chain length due to limited physical connectivity and a lower p value reduces edge density, which can improve embedding locality.

D. Minor-Embedding

A minor-embedding map assigns each logical qubit to a connected chain of physical qubits, ensuring that all logical edges are preserved. The Minorminer heuristic [3] is used to find embeddings that minimize chain length while satisfying topological constraints. The average chain length, L_{avg}, is used as a proxy for practical robustness: shorter chains are generally more robust to decoherence and control errors.

E. k-Hop Entropy Metric

Let $G = (V, E)$ be the physical graph. For a node u in V and k in N, the k-hop neighborhood is:

$$N_k(u) = \{v \in Vu\}: dist_{G_{list}}(u, v) \leq k\} \quad (1)$$

where $dist_{G_{list}}$ is the shortest-path distance evaluated on the distance reference graph (in practice, the embedding-induced subgraph). Distances were evaluated on the embedding-induced subgraph, whereas degrees were measured either on the same induced subgraph (induced) or on the full physical graph (full), as indicated. A degree-weighted distribution over $N_k(u)$ is defined with respect to a degree reference graph G_{deg}:

$$p_k^{(u)}(v) = \frac{deg_{G_{deg}}(v)}{\sum_{w \in N_k(u)} deg_{G_{deg}}(w)}, v \in N_k(u) \quad (2)$$

The k-hop entropy (base-2 Shannon entropy) is:

$$KHEM_k(u) = -\sum_{v \in N_k(u)} p_k^{(u)} log_2 p_k^{(u)} \quad (3)$$

For an embedded chain $C \subseteq V$, the chain-level aggregation is defined as:

$$KHEM_k(C) = \sum_{v \in C} w_v KHEM_k(v), \sum_{v \in C} w_v = 1 \quad (4)$$

with w_v chosen (e.g., uniformly) and reported for reproducibility. High KHEM values indicate a diverse local structure, while low values correspond to uniform or sparse neighborhoods. In the context of minor-embedding, KHEM captures how "structurally rich" the physical surroundings of an embedded chain are, which may correlate with robustness.

F. Jensen–Shannon Divergence Extension

To compare two k-hop distributions, such as between different embeddings or between neighborhoods with $k = 2$ and $k = 3$, the JSD [13] is:

$$JSD(P \parallel Q) = \frac{1}{2} KL(P \parallel M) + \frac{1}{2} KL(Q \parallel M) \quad (5)$$

where $M = \frac{P+Q}{2}$ and base-2 logarithms are used in KL, hence JSD$\in$ [0,1]. In this study, P and Q denote empirical distributions of per-node KHEM k values under two conditions (e.g., $k = 2$ versus $k = 3$). The choice of $k = 2$ and $k = 3$ builds directly on prior work [8], where sensitivity analyses on Chimera, Pegasus, and Zephyr topologies showed that $k = 1$ neighborhoods are too local to capture structural heterogeneity, $k = 2$ introduces meaningful cell-level differences, and $k = 3$ stabilizes cross-topology trends. These observations justify using $k = 2$ and $k = 3$ as the most informative levels for KHEM.

G. Robustness Prediction

The primary robustness indicators are: average chain length (L_{avg}), maximum chain length (L_{max}), mean hop distance between logical neighbors in the embedding, KHEM statistics, and JSD between k-hop distributions. Correlations between these metrics were examined to determine whether high KHEM or large JSD reliably predict embeddings with poor robustness characteristics, such as longer chains and greater hop distances.

III. RESULTS AND DISCUSSION

A. Regression Diagnostics and Out-of-Sample Validation

To examine collinearity, the Variance Inflation Factor (VIF) [17] was used, setting the thresholds at 5 and 10. The largest VIF was 63.54, three predictors exceeded 5 and two exceeded 10, indicating substantial collinearity among the KHEM-related predictors. Models were evaluated using grouped 5-fold cross-validation, ensuring that the observations from the same logical instance were not mixed within folds. Heteroskedasticity was assessed using a Breusch–Pagan test, which revealed significant heteroskedasticity in the augmented specifications ($p < 0.01$). Distributional assumptions were further evaluated using a QQ plot, as shown in Figure 1, which revealed minor deviations from normality that were deemed acceptable. The JSD (base-2) was computed from empirical histograms and used in two ways: induced versus full degree reference at a fixed k and $k = 2$ versus $k = 3$ under the same degree mode. In practice, lower JSD values indicate the limited incremental value of $k = 3$ relative to $k = 2$. Higher JSD values, on the other hand, suggest that $k = 3$ contributes an additional structural signal. The experimental evaluation was performed exclusively on a Zephyr-8 architecture. This choice allowed for a manageable computational load while preserving the key structural characteristics of larger Zephyr systems. The observed trends suggest broader applicability to larger configurations. Nevertheless, validation on higher-m architectures is still needed.

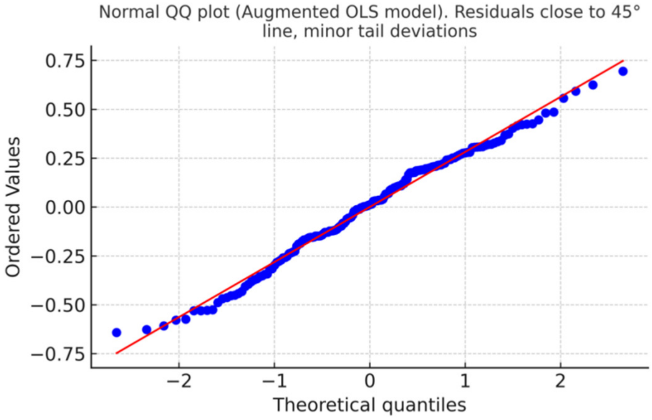

Fig. 1. Normal *QQ* plot: the residuals lie close to the 45° line, with only minor tail deviations, supporting the assumption of approximate normality.

B. Structured-Graph Experiments

To evaluate the generalizability of the findings beyond random and complete graphs, a small-scale study on two types of structured problem networks was conducted: 2D grids and 3-regular (Max-Cut-like) graphs embedded on Zephyr-8. The results confirmed that the main trends held true: mean hop distance remained a very strong predictor of average chain length ($R^2 \approx 0.97$), and KHEM with $k = 3$ captured additional variance ($R^2 \approx 0.30$). Additionally, the JSD between the distributions of $k = 2$ and $k = 3$ was consistently low (median ≈ 0.29), indicating that $k = 3$ yields a more stable signal. These preliminary results suggest that the predictive role of KHEM extends to structured problem families, despite varying in strength relative to hop-based measures.

C. KHEM Versus Chain Length

The performance and stability of quantum annealing embeddings depend on several factors, one of which is the average chain length—the number of physical qubits representing a single logical qubit in the minor embedding. Longer chains usually mean more resource-intensive configurations, which can lower solution quality and raise error rates. The relationship between KHEM, a measure of structural diversity in a node's k-hop neighborhood, and the mean hop distance between connected logical nodes as a function of the average chain length in D-Wave's Zephyr architecture is examined. The results for $k = 2$ and $k = 3$, as well as for hop distance, are compared using scatter plots with Ordinary Least Squares (OLS) regression [16]. The aim is to assess whether these metrics can reliably predict embedding cost and, consequently, reliability. Figures 2-4 present the relationships between key topological metrics (KHEM and mean hop distance) and the average chain length produced by the embedding process on D-Wave's Zephyr architecture. The analysis shows that KHEM with $k = 3$ exhibits a very strong positive correlation with chain length ($R^2 = 0.9837$), particularly in sparser logical graphs ($p = 0.10$). This indicates that higher entropy in the $k = 3$ neighborhood structure requires longer chains for embedding. This is consistent with the intuition that increased structural diversity creates more complex mapping demands on the physical topology.

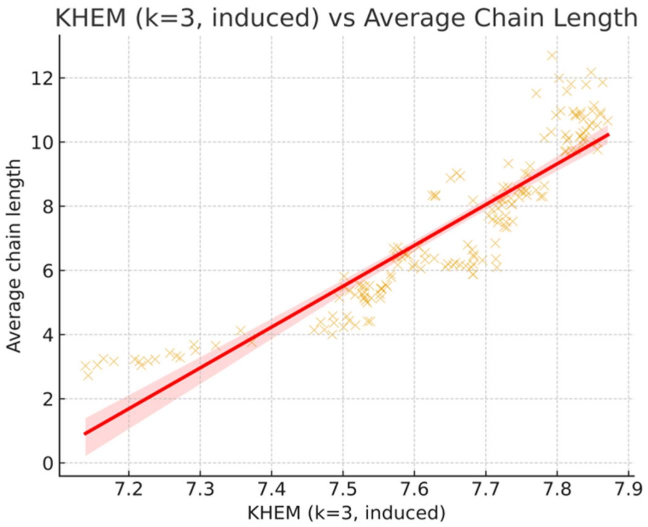

Fig. 2. Relationship between induced KHEM (k=3) and average chain length with OLS regression.

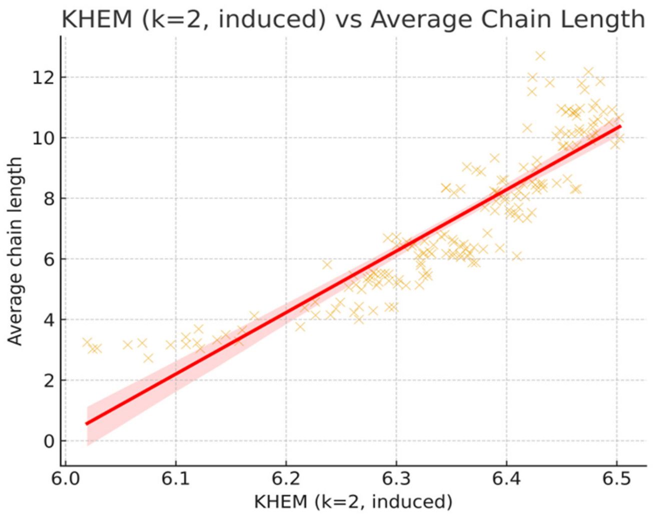

Fig. 3. Relationship between induced KHEM (k=2) and average chain length with OLS regression.

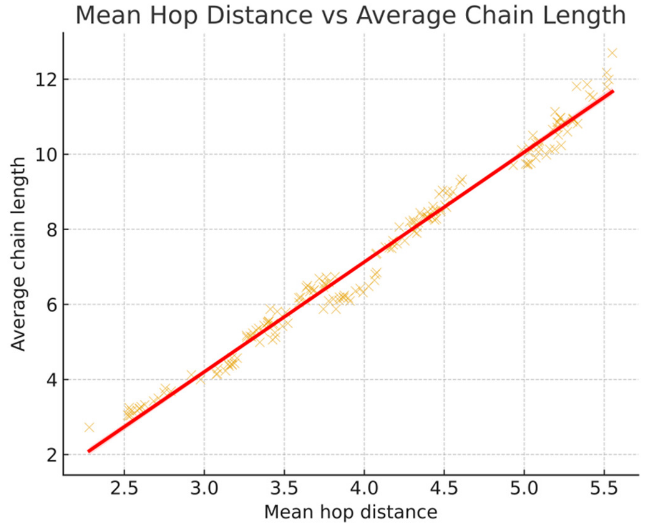

Fig. 4. Relationship between mean hop distance and average chain length with OLS regression.

Using KHEM with $k = 2$ results in a high correlation, though slightly lower than with $k = 3$. Regression models combining $k = 2$ KHEM and mean hop distance achieved the highest predictive accuracy ($R^2 = 0.9888$). This suggests that $k = 2$ captures complementary aspects of local connectivity that $k = 3$ alone may not fully reflect. The mean hop distance metric alone also demonstrates strong predictive capability ($R^2 = 0.9837$), confirming that spatial separation between logical qubits primarily drives chain length. Larger hop distances correspond to longer physical connections, which increases the likelihood of extended chains and impacts robustness. Overall, these findings validate that KHEM and mean hop distance are correlated with embedding cost and serve as potential predictors of embedding robustness. This information can be leveraged in embedding optimization and topology-aware algorithm design.

D. Hop Distance Distribution

In order to examine more the relationship between logical graph density and physical embedding characteristics, the distribution of hop distances for two extreme density configurations ($p = 0.10$ and $p = 0.25$) in ER logical graphs, was analyzed. Figure 5 displays these distributions, where for the sparse configuration ($p = 0.10$), the hop distance histogram is broader, indicating a higher percentage of long-distance connections between nodes. This suggests that the embedding process distributes logical edges more evenly across the physical topology, potentially reducing congestion and improving robustness. In contrast, for denser graphs ($p = 0.25$), the distribution of hop distances is narrower and skewed toward shorter distances. While short physical distances may seem advantageous initially, they often lead to multiple logical edges competing for the same physical qubits or couplers in practice. This localized congestion can increase chain lengths and reduce embedding stability under noise. Overall, these results align with the observed robustness trends: embeddings from sparser logical graphs tend to have more balanced hop distributions, whereas higher densities can introduce physical bottlenecks that negatively affect performance.

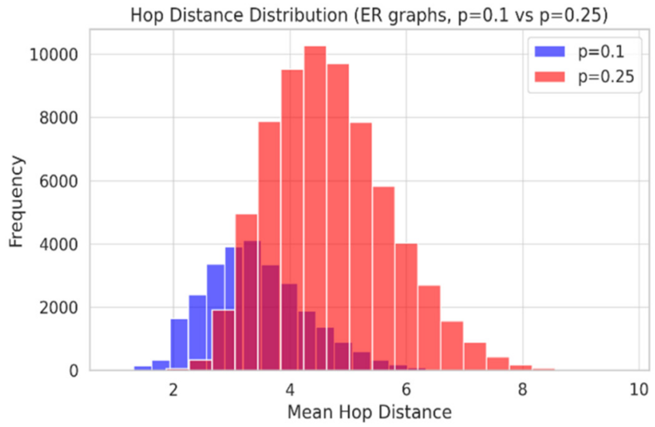

Fig. 5. Hop distance distributions for ER logical graphs with p = 0.10 (sparse) and p = 0.25 (dense). Sparse graphs show broader hop ranges; dense graphs concentrate on short hops, increasing congestion risk.

E. Robustness Prediction and JSD Analysis

The capability of KHEM to predict robustness in Zephyr architectures using JSD as a stability measure and regression model performance as an indirect indicator of robustness was evaluated in order to ascertain whether KHEM alone or in conjunction with other topological metrics can accurately forecast variations in chain length distributions under different logical graph densities. Figure 6 shows the JSD values for KHEM with k = 2 and k = 3. Lower JSD values indicate that the metric distribution remains more stable when subjected to noise or small-scale topological perturbations.

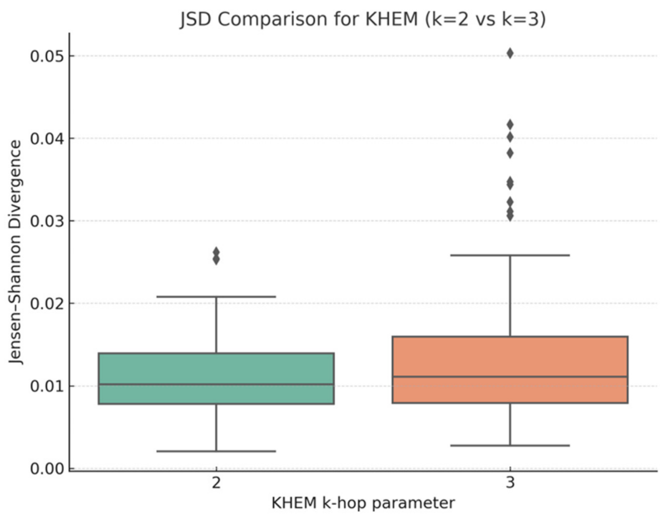

Fig. 6. JSD comparison for KHEM (k = 2 vs k = 3).

The results show that KHEM with k = 3 achieves lower JSD values than KHEM with k = 2, especially in sparse logical graphs (p = 0.10). This suggests that incorporating a larger k-hop neighborhood in the entropy computation increases the metric's robustness against structural fluctuations. While the JSD comparison highlights the stability advantage of k = 3 KHEM, regression models were also trained to predict the average chain length based on different metric combinations. The analysis revealed that combining KHEM with mean hop distance produces the highest R^2 scores, often exceeding 0.90 in sparse configurations. These results confirm that robustness prediction benefits from considering both a global entropy-based descriptor and a local connectivity measure.

F. Effect of Logical Graph Density

The KHEM-chain length relationship and the hop distance distributions both exhibit strong dependencies on the logical graph density (p). For sparse graphs (p = 0.10), the scatter plots show that KHEM values, particularly for k = 3, have a relatively linear and predictable relationship with average chain length. This pattern is confirmed by the hop distance distributions, which show more evenly distributed distances, reducing the likelihood of bottlenecks. However, when the density increases ($p \geq 0.25$), these patterns change significantly. KHEM distributions become compressed, hop distances cluster around lower values, and chain lengths increase disproportionately. This behavior is reflected further in the JSD analysis, where denser graphs tend to yield higher divergence under perturbations, indicating lower robustness. Overall, these results indicate that controlling the logical graph density is a practical way to improve the robustness of embeddings on Zephyr architectures and that maintaining a moderate density optimizes both chain length and stability.

G. Comparison with Previous Works

Previous studies on D-Wave architectures, have examined the structural descriptors of hardware graphs and their correlation with embedding complexity, determining the k-hop entropy as a sensitive measure of local degree heterogeneity and revealing systematic differences across the Chimera, Pegasus, and Zephyr families. However, these approaches were essentially descriptive and did not address predictive use cases. The present study formulates KHEM as a robustness predictor, by demonstrating statistically significant correlations with embedding cost indicators and by introducing JSD-based stability analysis between k = 2 and k = 3 horizons, and validating predictive accuracy through regression diagnostics and cross-validation. Unlike earlier descriptive analyses, the results of the present study show that:

- KHEM alone can serve as a robustness predictor; higher k-values (e.g., k = 3) yield stronger correlations, particularly in sparse logical graphs.
- Combined metrics (KHEM + hop distance) further improve predictive accuracy, achieving the highest R^2 scores in the experiments.
- Robustness under varying logical densities can be anticipated without full embedding simulations, offering practical benefits for algorithm–hardware matching.

These results represent a shift from static structural characterization to topology-aware robustness forecasting, making the method applicable not only to Zephyr, but also to future D-Wave topologies and other quantum annealing architectures.

H. Robustness and Threats to Validity

Beyond the baseline OLS models, regularized specifications (ridge and LASSO), polynomial terms, and 5-fold grouped cross-validation were examined. Collinearity was assessed using VIF and it was found that several KEM predictors exceeded conventional thresholds. In addition, testing for heteroskedasticity was performed via a Breusch–Pagan procedure. Robust (HC3) standard errors and residual diagnostics confirmed that these issues did not alter the direction or significance of the estimated effects. Overall, the results show that high in-sample R^2 values emerge partly due to redundancy among KHEM features. Yet, predictive accuracy remained strong under cross-validation, and KHEM with $k = 3$ provided a measurable improvement over hop distance alone. However, several limitations remain. Robustness was approximated using embedding proxies (average chain length and hop distance) rather than QPU outcomes (e.g., chain-break rates and solution quality). Additional QPU or noisy-simulator experiments are proposed to corroborate the proxy–outcome relationship. The experiments were restricted to a Zephyr-8 architecture and ER/complete logical graphs. Replication on larger Zephyr systems and across structured problem families is necessary. Minorminer parameter sensitivity (e.g., time limits, random seeds, and retries) may affect embeddings as well, suggesting that parameter sensitivity analyses with multiple runs should be conducted. Although this work's experiments with structured graphs were limited in scope, they confirmed that the observed trends extend to grid and Max-Cut families. This finding increases confidence that KHEM is not restricted to random topologies.

I. Interpretation of KHEM Predictive Power

KHEM's predictive power can be explained by its role as a proxy for degree heterogeneity in the hardware graph. Embeddings in regions of high heterogeneity require longer, more dispersed chains that are more vulnerable to errors. This observation aligns with empirical findings indicating a strong correlation between chain length, spatial dispersion, and higher chain break rates [21]. Benchmark studies on weak-strong cluster problems have further linked embedding geometry to performance degradation, supporting the interpretation that KHEM captures robustness-relevant structural effects [22]. While hop distance directly measures the geometric separation of logically adjacent variables, KHEM indirectly encodes the same effect by quantifying diversity in local degree distributions. This complementary mechanism explains why KHEM correlates with robustness proxies and why $k = 3$ networks, spanning multiple cells, provide the most stable signal.

IV. CONCLUSIONS

A topology-aware robustness prediction framework for Zephyr-8 using K-hop Entropy Metric (KHEM) as the primary descriptor, was examined. KHEM was found to be a reliable indicator, with $k = 3$ typically being stronger than $k = 2$, especially for sparse logical graphs. Meanwhile, hop-distance distributions provided complementary information on embedding constraints. In combination, KHEM and mean hop distance achieved the highest out-of-sample R^2, outperforming single-metric models. Logical graph density was decisive; lower densities produced shorter chains and more balanced hop distributions, indicating improved robustness. Compared to previous studies, this framework advances from a static topological characterization to a predictive, simulation-light methodology. This methodology facilitates algorithm–hardware matching and can be easily adapted to future Zephyr generations and other annealing architectures. Beyond its theoretical and empirical validation, KHEM can be incorporated directly into standard embedding workflows as a prefilter. A proposed pipeline is: generate candidate regions of the Zephyr topology (e.g., via Dwave-NetworkX), compute KHEM values ($k = 2$ and 3) for these regions, rank the candidates based on low mean KHEM and stable $k = 2$ versus $k = 3$ divergence, and restrict MinorMiner runs to the top-ranked subgraphs. This pre-filtering step reduces the number of embedding attempts on structurally fragile regions, which could improve embedding quality and reduce computational cost. In practice, a simple scoring function that combines mean hop distance and KHEM can be used to prioritize candidates.

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