Comparative Static Analysis in Reinforced Concrete Superstructure Beams: Working Stress vs. Limit State Approaches

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ABSTRACT

Historically, the design and analysis of bridge superstructures in Russia and Vietnam were based on standardized schemes utilizing the Working Stress Method (WSM). Although effective in its time, WSM is now considered outdated due to its conservative assumptions and limited ability to capture the actual performance of structural materials. With the advancement of structural design standards, the Limit State Method (LSM) has become the preferred analytical approach, offering a more realistic and comprehensive assessment of structural behavior. This study presents a comparative analysis of WSM and LSM applied to the static analysis of Reinforced Concrete beams in bridge superstructures. By recalculating existing components using LSM, the results indicate a significant reserve in load-bearing capacity, with an increase of approximately 30–32% for moving loads under the AK and NK loading schemes, values typically underestimated by WSM. These findings help explain the continued satisfactory performance of many aging bridge structures, originally designed for lower load classes, in meeting contemporary traffic demands.

Keywords-limit state method; working stress method; road bridge superstructure; finite element method; load carrying capacity

I. INTRODUCTION

Engineering design is fundamentally governed by the principles of safety, serviceability, and economy. Of these, the economic aspect entails balancing the cost of safety-enhancing measures with the potential financial losses associated with structural failure [1]. To achieve this balance, engineers rely on quantifiable representations of acceptable risk, typically expressed through Safety Factors (SFs) applied to loads, material strengths, and structural resistances within various design methodologies [1-5].

Bridges represent critical links in transportation infrastructure, playing an essential role in facilitating logistics, enabling economic development, and supporting regional connectivity. Ensuring their long-term structural integrity is vital for public safety and network reliability. In rapidly urbanizing countries, such as Vietnam, increased traffic volumes and higher axle loads have placed significant stress on aging bridge structures. To address these challenges, extensive research in Vietnam has been devoted to structural health monitoring and computational performance assessment of bridges. For example, authors in [6] employed Linear Variable Differential Transformer (LVDT) sensors to monitor displacements in an urban railway bridge. Additionally, the application of the Finite Element Method (FEM) has become increasingly prominent in evaluating bridge behavior. A comparative study [7] demonstrated the advantages of FEM over traditional analytical approaches for continuous RC bridges, particularly in terms of accuracy and adaptability.

Statistical analysis of the transportation trends over the past 50 years reveals a 30–50% increase in vehicle numbers, accompanied by rising vehicle weights and road usage intensity. The equivalent wheel load on road surfaces has increased by approximately 25–30% compared to rail-based transport systems [8], primarily due to the expansion of private vehicle ownership, freight logistics, and urban development.

Many existing bridges in Vietnam and post-Soviet countries were constructed according to outdated design standards that no longer reflect modern performance expectations. These structures were commonly designed using WSM, which relies on linear elastic behavior and permissible stress limits. Today's traffic loads often exceed those assumed in the original design, raising questions about the sufficiency of these structures' remaining load-bearing capacity. Moreover, bridges play a vital role in emergency response and disaster mitigation; hence, their structural resilience under extreme conditions, such as earthquakes or floods, must be re-evaluated. Figure 1 illustrates the comparative diagrams of the relationship between the equivalent standard strip load and span length for distributed automobile loads and for concentrated heavy single loads.

(a)

(b)

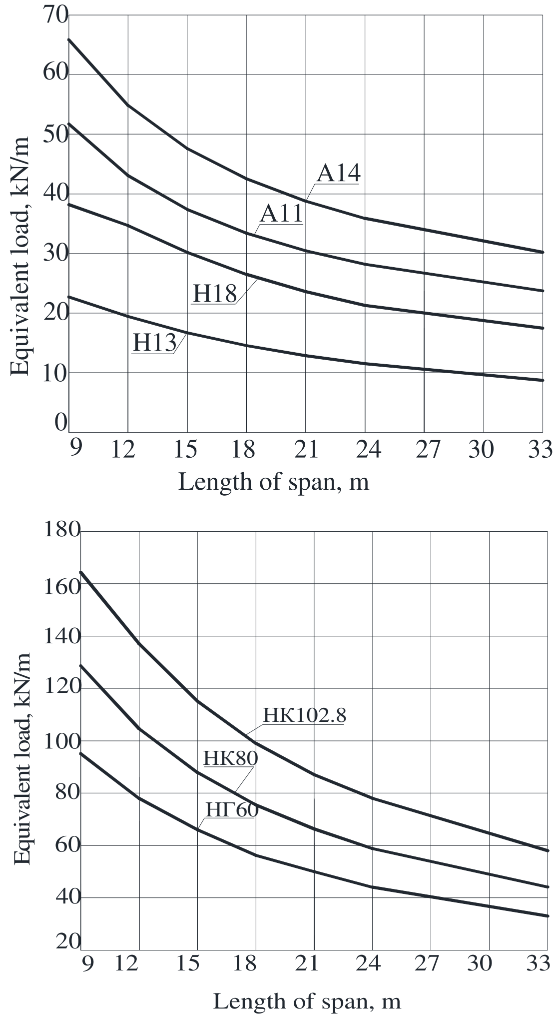

Fig. 1. Comparative diagrams showing the relationship between the equivalent standard strip load and span length: (a) for distributed automobile loads, (b) for concentrated heavy single loads.

The subject of this study is an RC beam bridge, originally designed in accordance with the "Issue 56" standard from the 1960s–1970s. These bridges feature precast T-section girders, in-situ cast diaphragms, and a monolithic concrete deck. Their design was governed by standard lane loads N13 and N18 using the WSM. This type of superstructure was widely utilized due to its industrialized construction method, particularly during post-war reconstruction in Vietnam and the former Soviet Union [6, 8].

Many such bridges are still operational today, having undergone routine maintenance and localized upgrades. However, they now face significantly higher live loads than originally anticipated. For this reason, their structural capacity must be re-evaluated using modern techniques and updated load models [9-10]. Figure 2 presents the typical cross-section of the superstructure, and Table I summarizes the key geometric and material properties.

(a)

(b)

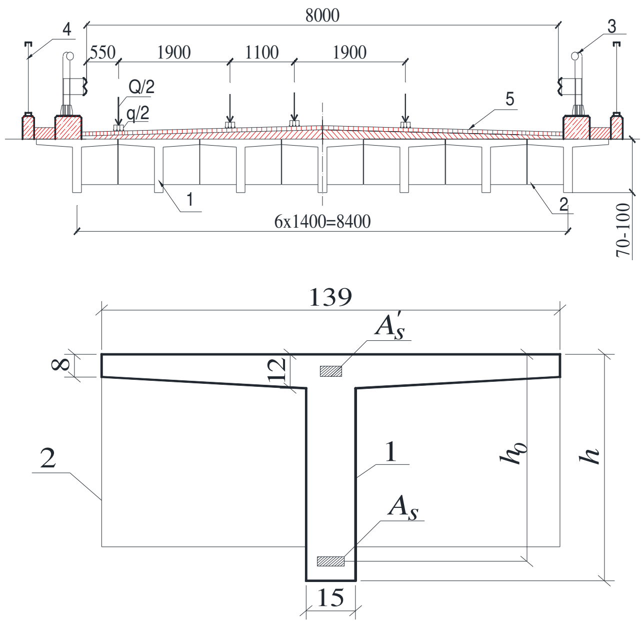

Fig. 2. Cross-sections: (a) of the superstructure according to "Issue 56", (b) of a typical beam. 1 – T-section beam with conventional reinforcement, 2 – diaphragms, 3 – RC curbs with metal railings, 4 – metal barrier fences, 5 – multi-layer road surface.

To evaluate the residual load-bearing capacity under current traffic demands, the bridge was reanalyzed using the LSM principles along with FEM-based software tools: SAP2000. The modeling procedure followed consistent parameters across all platforms. Authors in [8] review international and Russian practices in bridge strength monitoring, emphasizing its role in enhancing structural safety and reliability. Previous work, such as [9, 10], compared WSM and LSM for RC slab bridges and found that LSM-based assessments generally yield 30–35% higher load-bearing capacities. Although degradation and aging effects were not considered, empirical field surveys indicate that most beams in these bridge types remain in serviceable condition due to consistent maintenance protocols.

II. ANALYSIS METHODS FOR RC BEAMS IN BRIDGE SUPERSTRUCTURES

Design guidelines and SFs, consolidated within standardized codes, play a critical role in ensuring structural reliability. These codes support informed engineering decisions by balancing safety, functionality, and cost-effectiveness. In Russia, during the modern era of RC span structure development (beginning in 1960), four successive versions of design standards were implemented: those from 1948, 1962, 1984, and 1991 [11–16]. Until 1962, structural calculations were conducted using WSM. The adoption of the technical specifications of SN 200-62 [15] marked the transition to LSM. Under WSM, a uniform SF was applied to all loads and materials, promoting a conservative design philosophy. Internal forces were first determined, and the resulting stresses were then compared to allowable stress limits, which were derived by dividing the material strength by prescribed SFs.

While straightforward, this method did not account for variations in loading conditions or failure modes, often leading to overdesign and inefficient use of materials. In contrast, LSM differentiates between various failure modes, assigning SFs based on the probability and severity of failure. This allows for a more optimized and material-efficient design. LSM replaces the single SF with three groups of reliability factors: γ_f for loads, γ_m for materials, and γ_n for structural importance, along with additional factors for working conditions and load combinations. The allowable stresses (σ_a) for reinforcement are typically taken as 0.5 of the yield strength σ_T, whereas modern design resistances are determined as:

$$R_s = \gamma_m \times \sigma_T \quad (1)$$

where $\gamma_m = 0.9$.

Table I provides a comparison between the (σ_a) from the design standards used before 1962 and the modern design resistances R_s for the same reinforcement class.

TABLE I. COMPARISON OF ALLOWABLE STRESSES AND R_S

Class (grade) of reinforcement	Unit	(σ_a)	R_S
AI (St. 3)	MPa	125	210
AII (St. 5)	MPa	150	265

A. *Working Stress Method*

For centuries, civil engineering design was based on the common sense, judgment, and experience of the engineer, along with trial and error. WSM was first developed in the discipline of structural engineering because of the need to replace the traditional method of trial and error with something more "scientific". It was built on Newton's laws of motion, and the theory of elasticity, which were the only tools available at the time for structural design [17]. The basis of the structural WSM is to ensure that the induced stresses are less than the allowable stresses throughout the structure when it is subjected to the "working" or service load [1]. The concept is the same for the bridge superstructures. Thus, it is essential that the engineers use their judgment and experience. In fact, the factors of safety were developed as a result of experience, trial and error, and insights gained from previous designs. The global SF represents a relationship between allowable and applied quantities. FS can be defined as the ratio of the resistance of the structure (capacity) to the load effects acting on the structure (demand). However, the WSM shows certain disadvantages. These include [1]:

- It does not encourage the engineer to think about and differentiate between the behavior of the structure under ultimate loading and serviceability conditions.
- It is largely deterministic and does not lend itself to probabilistic assessments of safety level. This design method provides only an implicit indication of probability of failure because the global FS has been derived from experience.

Despite these limitations, the WSM has proven to be a useful tool in bridge design, and has been the traditional design method for over 100 years. The accumulation of experience from the years of deploying the particular method has been recognized, and thus the global FS has been used to calibrate the more recent limit state methods and factors.

B. *Limit State Method*

There is a trend toward LSM in bridge design. The motivation for this trend is to improve design compatibility between structural engineering and also improve the economy and safety of the designs. Limit states define the various ways in which a structure fails to satisfy two basic requirements: safety from collapse, and satisfactory performance of the structure for its intended use [17]. When a structure (or a component of a structure) fails to satisfy one of its intended performance criteria, it is said to have reached a limit state [1-2]. The basic concept of LSM is that the resistance of a structure be greater than the load effects. Measures of safety are often incorporated into this type of design through the use of partial factors. In this approach, the specified or characteristic loads are multiplied by their respective partial factors to arrive at the design strength parameters for the calculation of resistance. Partial factors are obtained by calibration with conventional WSM and reliability analysis. The partial SFs were first selected to provide designs similar to those obtained by WSM using traditional total SFs. As a result, structures designed using WSM tend to have a relatively large reserve of load-bearing capacity.

III. ANALYSIS FOR RC BEAMS OF ROAD BRIDGE SUPERSTRUCTURES

The road bridge superstructures, designed according to the "Issue 56" model, are available in two versions to accommodate different load requirements. These versions correspond to load classifications N13, NG-60, and N18, NK-80, differing in both the strength grade of the concrete and the cross-section of the load-bearing steel elements. Table II presents the key technical parameters of the beam, including the section height and the cross-sectional area of reinforcement at mid-span and at the quarter-span positions. Beam material: with RC superstructures designed for N18 and NK-80 loads using concrete of M300 (B22.5), and RC superstructures designed for N13 and NG-60 loads using concrete of M250 (B19).

TABLE II. TECHNICAL PARAMETERS OF TYPICAL BEAMS FOR DESIGN MODEL "ISSUE 56"

Parameter	Location	Beams designed for	Calculated parameters for beam length		
			11.36	14.06	16.6
Calculated span, L (m)	–	–	11.1	13.7	16.3
Beam depth, h (cm)	–	–	80	85	100
Effective height, h₀ (cm)	At mid-span	–	70.3	74.8	89.3
–	At quarter-span	–	72.3	75.7	90.7
Reinforcement area, A_s (cm²)	At mid-span	N13/NG-60 loads	44.2	52.3	64.3
–	At quarter-span	N13/NG-60 loads	36.2	48.2	48.2
–	At mid-span	N18/NK-80 loads	52.3	72.4	76.4
–	At quarter-span	N18/NK-80 loads	48.2	64.3	64.3

Figures 3(a) and (b) illustrate an example of longitudinal reinforcement for 11.36 and 14.06-meter-long beams in a road bridge superstructure. In the original design (1957), the span width was G7 + 2 × 0.75. After reconstruction to meet modern standards, the span width was increased to G8 + 2 × 0.75. The analysis and calculations of the studied structures show that the determining factor influencing the design approach and load-bearing capacity assessment is the verification of strength based on the bending moment values. In calculations using the allowable stress method, this verification is also crucial in determining the cross-sectional dimensions of the beam and the cross-sectional area of the longitudinal reinforcement. The design and installation of beam reinforcement according to standard design principles are developed in such a way that, when the full load-bearing capacity for the design moment is utilized, the reserve strength for further checks according to the first and second limit states, as per Russian construction code [10], is preserved (under the same vertical design load).

Design material properties: deformation modulus of concrete (class B22.5) Eb = 25.650 MPa, reinforcement (class AII) Es = 206.000 MPa, design strength of concrete Rb = 11.75 MPa, reinforcement Rs = 236 MPa. When determining the bending moment in beams subjected to temporary loads, spatial calculations were performed using two methods: the method in [17] and FEM, employing the third and fourth calculation schemes from [4, 5].

The results of these spatial calculations, along with the diagrams depicted in Figure 4, indicate that assessing the bearing capacity of the beams does not require comparing the calculated and allowable moments along the entire beam length. Instead, it is sufficient to compare the corresponding bending moments at the mid-span.

Therefore, the subsequent analysis focused on the mid-span cross-sections of the span structures and the most heavily loaded beams (located at x=l/2). Figure 4 presents an example of the distribution diagram for the bending moment ratio (moment distribution coefficient) M_d, which represents the bending moment acting on individual beams (from the first to the seventh) relative to the total calculated bending moment M_{tt} at the mid-span. This analysis was conducted for a structure with 11.36 m and 14.06m spans and overall dimensions of G8+2×0.75, subjected to two temporary vertical load strips (AK) approaching the left barrier (corresponding to the "second case" load described in Section 2.12 of [18].

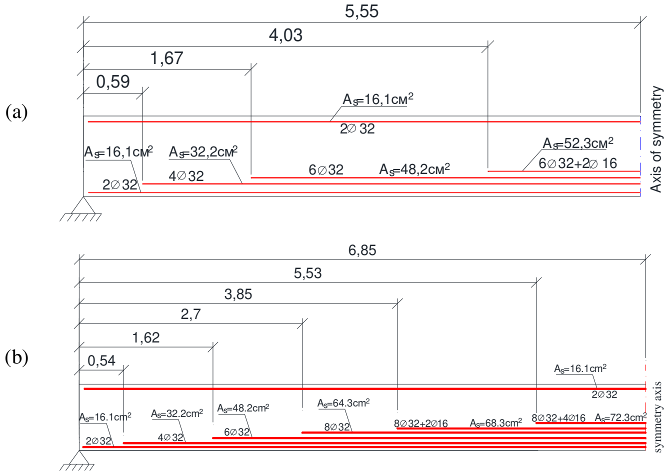

Fig. 3. Reinforcement scheme along the length of the beam from "Issue 56": (a) with a length of 11.36 m, (b) with a length of 14.06 m.

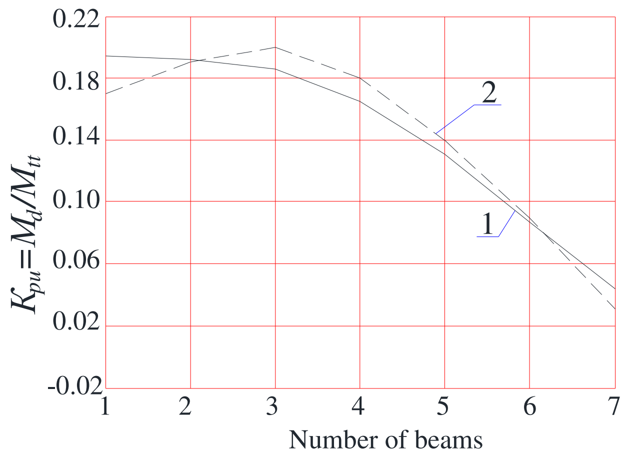

Fig. 4. Distribution coefficient chart of bending moment $k_{pu}=M_d/M_{tt}$ at the mid-span section of superstructure, with a span length of 11.36 m and a width of G8+2×0.75: 1- according to [18], 2 - according to FEM.

Figures 5 and 6 present the "ultimate" and "allowable" bending moment and shear force diagrams for 11.36 m and 14.06m -long beams, determined using LSM and the allowable stress method, respectively. These diagrams are based on the distribution of working reinforcement according to standard design practices. Additionally, Table III compares the ultimate and allowable bending moments at the mid-span and quarter-span of the beam for two steel options, each corresponding to a different type of load. The data from Figures 5, 6, and Table III indicate that the maximum bending moment calculated using LSM is 1.6 times greater than the corresponding allowable bending moment obtained from the allowable stress method.

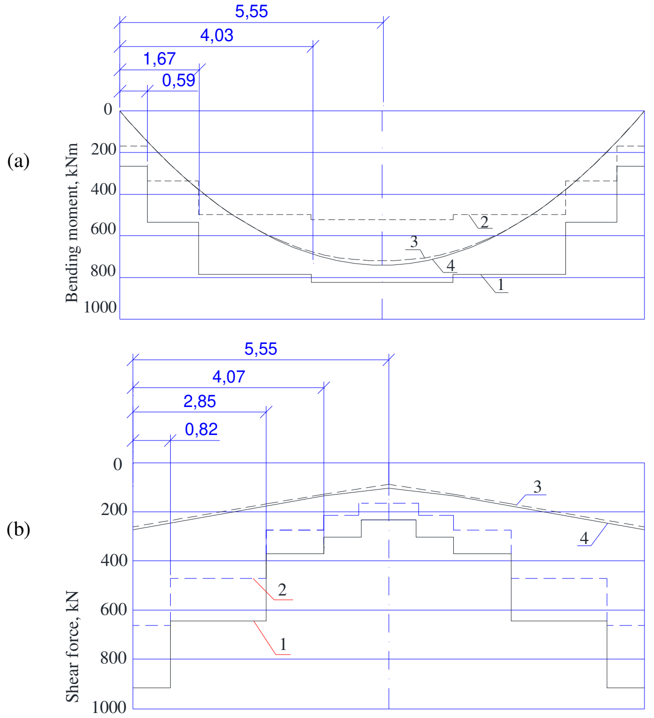

Fig. 5. Diagrams for a typical beam with a length of 11.36 m, reinforced for loads H-18 and NK-80: (a) bending moments, (b) shear forces. 1 – "ultimate" according to LSM, 2 – "allowable" according to WSM, 3 – calculated moments and shear forces from load A11 and permanent load at K_{pu}=0.22, 4 – from load NK-80 and permanent load at K_{pu}=0.25.

In the system of standardization for bridge maintenance and operation, load-bearing capacity indicators are used, defined as moving load classes according to the AK and NK schemes that the bridge structure can withstand. During the certification and inspection of bridges, the load-bearing capacity is assessed by calculating the strength of the most critical and heavily loaded element, taking into account existing defects.

For RC superstructures, this calculation involves checking the strength of normal sections under the bending moment at the mid-span section of the most heavily loaded or damaged beam.

In accordance with these principles, the load-bearing capacity indicators for AK and NK moving loads are determined by:

$$AK = \left(\frac{M_{ult} - M_{ultLSM}}{M_{A11}}\right) \times 11 \quad (2)$$

$$AK = \left(\frac{M_{ult} - M_{ultLSM}}{M_{NK80}}\right) \times 11 \quad (3)$$

where M_{ult} is the ultimate bending moment in the calculated section of the beam, M_{st_LSM} is the bending moment from static loads calculated according to LSM, M_{st_WSM} is the bending moment from static loads calculated according to WSM, and M_{A11} and M_{NK80} are the bending moments in the most heavily loaded beam from moving loads A11 and NK-80.

The numbers 11 and 80 indicate the class of vehicle load A11 and the weight of a single NK-80 vehicle.

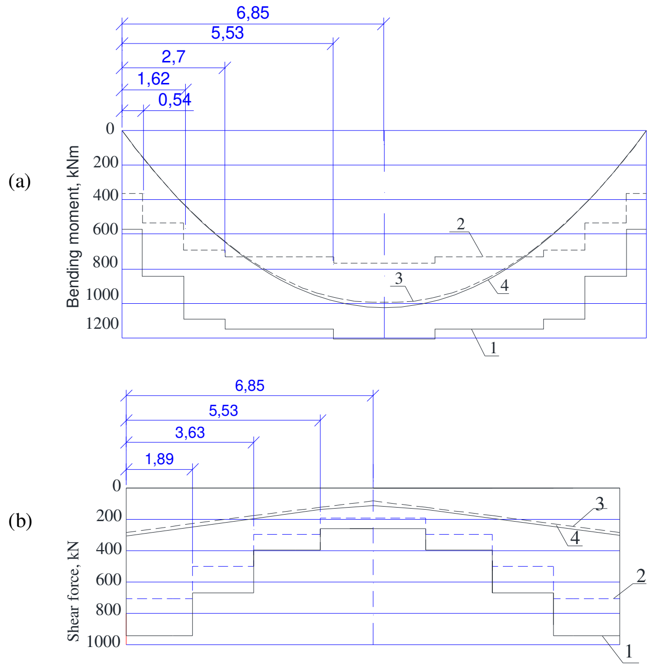

Fig. 6. Diagrams for a typical beam with a length of 14.06 m reinforced for loads H-18 and NK-80: a – bending moments, b – shear forces, 1 – "ultimate" according to the LSM, 2 – "allowable" according to the WSM, 3 – calculated moments and shear forces from load A11 and permanent load at K_{pu}=0.22, 4 – from load NK-80 and permanent load at K_{pu}=0.25.

The results presented in Figures 5 and 6, along with the data in Table III, indicate that recalculating beams using LSM (in place of the Allowable Stress Method employed in the original 1960s design) and adopting a more flexible analytical approach reveal a significant reserve in the load-bearing capacity of these bridge span structures. Despite their age, these structures remain capable of safely carrying current traffic loads, confirming the robustness of their original design and construction.

TABLE III. BENDING MOMENTS AND CAPACITY INDICATORS FOR G8–G11.5 SUPERSTRUCTURES

Name of parameters	Beam length (m)			
	11.36	11.36	14.06	14.06
Cross-sectional area of reinforcement, As (cm^2)	44.23	52.26	52.26	72.36
Ultimate bending moment (LSM), Mult (kNm)	736	825	931	1202
Allowable bending moment (WSM), Mallow (kNm)	461	525	579	766
Bending moment from static load (kNm)				
According to LSM, M_{st_LSM}	235	235	362	362
According to WSM, M_{st_WSM}	196	196	303	303
M_{ult}-M_{st}_LSM (kNm)	501	590	569	840
M_{allow}-M_{st}_WSM	265	329	276	463
Bending moment from live load (kNm)				
With A11 load (K_{pu} = 0.224)	492	492	629	629
With NK 80 load (K_{pu} = 0.27)	507	507	659	659
Design Load Class				
$AK = \left(\frac{M_{ult} - M_{ultLSM}}{M_{A11}}\right) \times 11$	A11,2	A13,3	A10	A14,7
$AK = \left(\frac{M_{ult} - M_{ultLSM}}{M_{NK80}}\right) \times 80$	NK79	NK93	NK69	NK102

Due to the increased calculated load-bearing capacity (compared to the allowable capacity) of concrete and reinforcement, the load-bearing capacity of the bridge span system for live loads has increased by 30–32%. This proves that the load-bearing capacity of bridge span structures initially designed for load class N18 (N13) is still fully capable of withstanding current loads of type A14 (A11).

IV. CONCLUSION

This study has demonstrated that re-evaluating existing Reinforced Concrete (RC) beam bridges originally designed using the Working Stress Method (WSM) through the Limit State Method (LSM) and modern spatial analysis techniques can uncover significant reserve load-bearing capacity. Specifically, recalculations showed a 30–32% increase in capacity under AK and NK moving load schemes, enabling many bridges designed for outdated H18 (H13) and NK80 (NG60) load classes to meet the current A14 (A11) and NK-100 (80) design requirements. For practicing engineers, these findings suggest that, before undertaking major retrofits or replacements, aging bridge structures—especially those built under the "Issue 56" standard—should be reassessed using LSM and modern Finite Element Method (FEM) tools, potentially confirming their continued adequacy and extending service life in a cost-effective manner.

However, the study is subject to limitations. The analysis was based on idealized FEM models that assume uniform material properties, simplified boundary conditions, and default software parameters. The effects of long-term deterioration, fatigue, or environmental degradation were not explicitly modeled. Future research should focus on integrating deterioration modeling, probabilistic analysis of material and load uncertainties, and the use of real monitoring data for model validation.

In summary, this research highlights the potential of modern analytical methods to unlock hidden structural capacity in aging bridge infrastructure and encourages a transition from conservative legacy approaches to performance-based, data-driven evaluation practices aligned with contemporary engineering standards.

COMPETING INTERESTS

Authors declare that there is no conflict of interests regarding the publication of this paper.

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